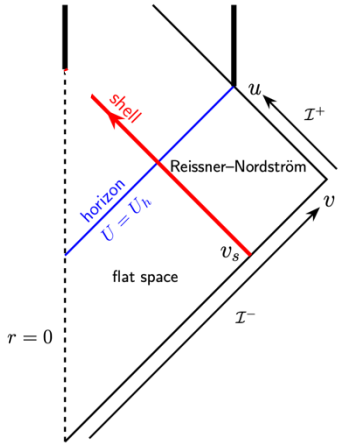
Enhanced Correlations in Hawking Radiation from Near-Extremal Collapse

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- Based on 2607.12989 with S. Bintanja and S. Kim

Introduction



Collapse of spherical charged shell in asympt flat space to form a RN black hole (Vaidya solution)

Adding a massless scalar, we should be able to compute what comes out order-by-order in G (expect info loss!). Hawking gave answer in free field limit.

Near extremality, when $T\frac{\partial T}{\partial M} \sim T$ thermal fluctuations become large [Preskill, Schwarz, Shapere, Trivedi, Wilczek 1991]

Fixed classical background breaks down near horizon: fluctuations governed by effective coupling $g \sim \frac{G}{r_0^3 T_H}$. Quantum gravity relevant at low curvature.

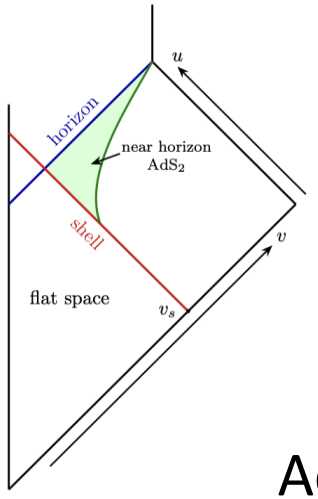
Much activity over the past decade based on AdS_2, Schwarzian QM, etc. Not immediately applicable to full asymptotically flat collapse scenario.

Here I consider this problem using "old school" methods. Goal is to compute scalar field correlators on $\mathcal{I}^+$

JT and Schwarzian

Progress over past decade based on near–horizon Schwarzian QM

[Almheiri, Polchinski
Jensen
Engelsoy, Mertens, Verlinde;
Maldacena, Stanford, Yang
Iliesiu, Turiaci, …]
[Brown,Iliesiu,Penington, Usatyuk
Biggs
Emparan
Betzios,Papadoulaki, Zhou
Luo,Pando-Zayas, …]



Strong coupling effects confined to AdS2 region. Solve theory in this region and then match to external world

$$ds_4^2 = ds_2^2 + \Phi^2(t,r)d\Omega^2 \ , \quad \Phi = r_0(1+\phi)$$

Action: $$S = S_0 + \frac{r_0^2}{G}\int d^2x\sqrt{g_2}\phi(R_2 - \Lambda_2) + \underbrace{\frac{r_0^2}{G}\int_{bndy} dt\phi K_2}_{\text{JT gravity}} + O(\phi^2)$$

Boundary curve: $\tau(u)$

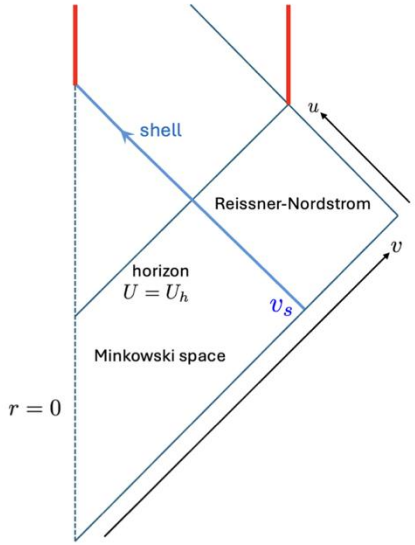


[Maldacena, Stanford, Yang]

Euclidean action: $S = C\int du \mathrm{Sch}[\tau(u)]$

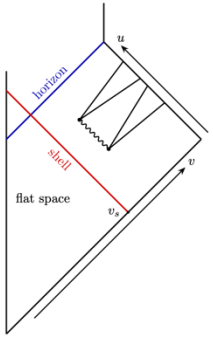
Questions: role of dilaton as Lagrange multiplier; Euclidean versus Lorentzian; …

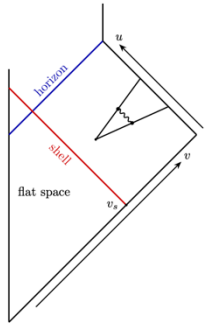
Collapse problem



Consider no incoming radiation (Unruh state).

Want in-in expectation values of observables at future null infinity.





- D = 3+1 Einstein-Maxwell + charged shell + massless neutral scalar field
- Restrict to s-wave (captures strong coupling effects)
- Treat scalar as free in flat region inside shell
- Do not quantize shell: use it to set initial data for scalar field outside shell

s-wave Hamiltonian

Build on earlier work [Berger, Chitre, Moncrief, Nutku; Unruh]

s-wave reduction: $ds^2 = -N^2(t,r)dt^2 + L^2(t,r)\left(dr + N^r(t,r)dt\right)^2 + R^2(t,r)d\Omega^2$

$$A = A_t(t,r)dt\ , \quad \phi = \phi(t,r)$$

Action: $S = \int d^2x \left(\pi_\phi \dot{\phi} + \pi_L \dot{L} + \pi_R \dot{R} - N\mathcal{H}_t - N^r \mathcal{H}_r\right) - \int_{bndy} dt H_{\rm ADM}$

constraints: $\mathcal{H}_t^\phi + \mathcal{H}_t^G = 0, \quad \mathcal{H}_r^\phi + \mathcal{H}_r^G = 0.$

$$\mathcal{H}_t^\phi = \frac{1}{2}\left(\frac{\pi_\phi^2}{LR^2} + \frac{R^2}{L}\phi'^2\right), \quad \mathcal{H}_r^\phi = \pi_\phi \phi'$$

from Gauss law constraint

$$\mathcal{H}_t^G = \frac{GL\pi_L^2}{2R^2} - \frac{G\pi_L\pi_R}{R} + \frac{1}{G}\left[\left(\frac{RR'}{L}\right)' - \frac{R'^2}{2L} - \frac{L}{2}\right] + \frac{Q^2 L}{2R^2}, \quad \mathcal{H}_r^G = R'\pi_R - L\pi_L'$$

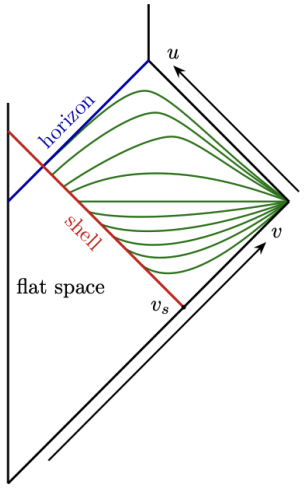
S

Strategy: choose gauge; solve constraints; substitute back into action after deriving expression for $M_{\rm ADM}$ by demanding good variational principle [Regge,Teitelboim]

No use of equations of motion! Metric variables determined by (π_ϕ, ϕ)

Choose gauge, solve constraints

Only need to cover region outside horizon, so we can choose Schwarzschild type gauge



$$ds^2 = -N^2(t,r)dt^2 + L^2(t,r)\left(dr + N^r(t,r)dt\right)^2 + R^2(t,r)d\Omega^2$$

Gauge choice: $\pi_L = 0, \quad R = r$

Constraints can be solved explicitly: $\frac{1}{L^2(r)} = f(r) - \frac{2G}{r}\int_{r_s}^{r} dr' f(r') h(r') e^{-\int_{r'}^{r} dr'' \frac{2Gh(r'')}{r''}}$

$$f(r) = 1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}, \quad h(r) = \frac{1}{2}\left(\frac{\pi_\phi^2}{r^2} + r^2\phi'^2\right)$$

ADM energy: $H_{\rm ADM} = \lim_{r\to\infty} \mathcal{M}(r)$ $\qquad \mathcal{M} = \frac{G\pi_L^2}{2R} + \frac{R}{2G}\left[1 - \left(\frac{R'}{L}\right)^2\right] + \frac{Q^2}{2R} \to \frac{r}{2G}\left[1 - \left(\frac{1}{L}\right)^2\right] + \frac{Q^2}{2r}$

Constraint solution gives:

$$H_{\rm ADM} = M + \int_{r_s}^{\infty} dr' f(r') h(r') e^{-\int_{r'}^{\infty} dr'' \frac{2Gh(r'')}{r''}}$$

Reduced action

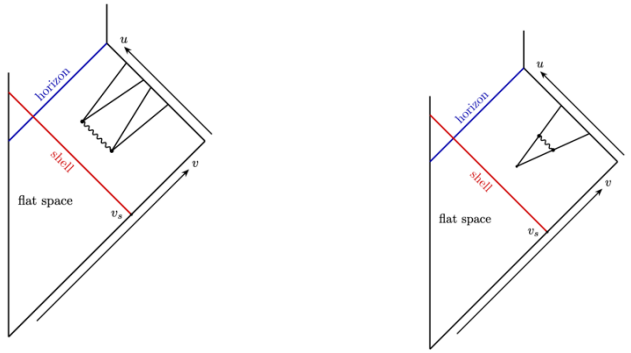
Reduced scalar action capturing s-wave gravity interactions

$$S = \int dt \int_{r_s}^{\infty} dr \left(\pi_\phi \dot{\phi} - f(r) h(r) e^{-\int_r^\infty dr' \frac{2Gh(r')}{r'}} \right) \qquad f(r) = 1 - \frac{2GM}{r} + \frac{GQ^2}{r^2} \,, \quad h(r) = \frac{1}{2}\left(\frac{\pi_\phi^2}{r^2} + r^2 \phi'^2 \right)$$

Integrate out π_ϕ: $\quad S = \int dt \int_{r_s}^{\infty} dr \left[\frac{1}{2} r^2 \left(\frac{1}{f} \dot{\phi}^2 - f \phi'^2 \right) - 2G f(r) h(r) \int_r^\infty dr' \frac{h(r')}{r'} \right] + O(G^2)$

Nonlocal in space, but local in time

Can use standard perturbative methods to compute correlations in outgoing Hawking radiation



Vertices can be anywhere, but enhanced effects occur in near-horizon region

Near-horizon interactions

Examine interactions near horizon in near-extremal regime

Near-extremal scaling: $r_\pm = r_0 \pm \lambda\rho_h \qquad \lambda \to 0 \qquad r_\pm = GM \pm \sqrt{G^2M^2 - GQ^2}$

Near-horizon region: $r = r_0 + \lambda\rho_h \coth z$

After rescaling other variables: $ds^2 \to r_0^2\left(\frac{-d\tilde{t}^2 + dz^2}{\sinh^2 z} + d\Omega^2\right) + \ldots = \mathrm{AdS}_2 \times S^2$

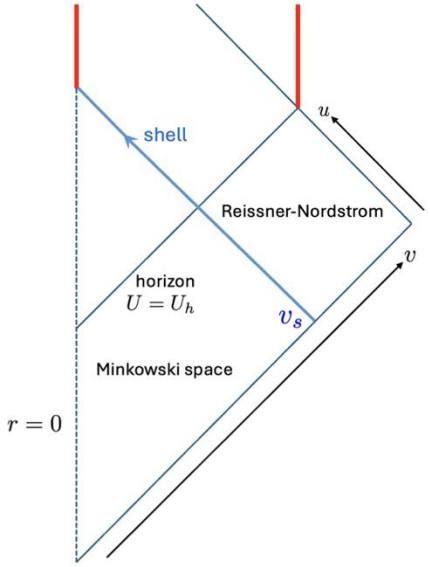
$$T_H = \frac{r_+ - r_-}{4\pi r_+^2} \approx \frac{\rho_h}{2\pi r_0^2}\lambda.$$

Near-horizon action: $S = \int dt \int_0^\infty dz \left[\tilde{\pi}_\phi \dot{\tilde{\phi}} - \frac{1}{2}\left(\tilde{\pi}_\phi^2 + \tilde{\phi}'^2\right) e^{-\frac{1}{2}g\int_0^z dz' \sinh^2 z' \left(\tilde{\pi}_\phi^2 + \tilde{\phi}'^2\right)}\right]$

Dimensionless coupling: $g = \frac{2G}{\rho_h r_0 \lambda} \approx \frac{G}{\pi r_0^3 T_H}.$ Low temperature -> strong coupling

Large flucs in (ϕ, π_ϕ) translate via constraints to large flucs in L → throat length

Free field case



inside shell: (U, V)

outside shell: (u, v)

Matching at shell: $u = U - \frac{2r_+^2}{r_+ - r_-} \ln\left(\frac{v_s - U}{2} - r_+\right) + \frac{2r_-^2}{r_+ - r_-} \ln\left(\frac{v_s - U}{2} - r_-\right)$

$v = V$

State emerging from shell is (U,v) vacuum (Unruh vacuum)

In free field theory, ignoring greybody factors: $\left\langle \partial_U \tilde{\phi}\,(U_1, v_1)\, \partial_U \tilde{\phi}\,(U_2, v_2) \right\rangle = -\frac{1}{4\pi} \frac{1}{(U_1 - U_2 - i\varepsilon)^2}$

$$\left\langle \partial_v \tilde{\phi}\,(U_1, v_1)\, \partial_v \tilde{\phi}\,(U_2, v_2) \right\rangle = -\frac{1}{4\pi} \frac{1}{(v_1 - v_2 - i\varepsilon)^2}$$

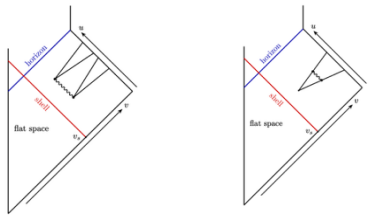
→ $\langle T_{uu} \rangle = -\frac{1}{2} \lim_{U_2 \to U_1} \left[\frac{\left(\frac{\partial u_2}{\partial U_2} \frac{\partial u_1}{\partial U_1}\right)^{-1}}{(U_2 - U_1 - i\varepsilon)^2} - \frac{1}{(u_2 - u_1 - i\varepsilon)^2} \right] = \frac{1}{12} \left(\frac{\partial u_1}{\partial U_1}\right)^{-2} \mathrm{Sch}\,(u_1, U_1) \approx \frac{(r_+ - r_-)^2}{96 r_+^4} = \frac{\pi^2 T_H^2}{6}$ as $u \to \infty$

standard result

Perturbation theory

Perturbative action: $$S = \int dt \int_{r_s}^{\infty} dr \left[\frac{1}{2} r^2 \left(\frac{1}{f} \dot{\phi}^2 - f \phi'^2 \right) - 2G f(r) h(r) \int_r^{\infty} dr' \frac{h(r')}{r'} \right] + O(G^2)$$

Sufficient for computing:



Full computation "straightforward" but messy due to non-universal early time behavior, grey-body factors.

Focus on late-time correlators: restrict vertices to near-horizon region, and include grey-body factors at the end.

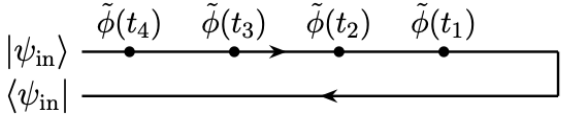
Near-horizon action (dropping tildes) $$S[\phi] = \frac{1}{2} \int dt \int_0^{\infty} dz \left(\dot{\phi}^2 - \phi'^2 \right) + g \int dt \int_0^{\infty} dz \int_0^{z} dz' \sinh^2 z' T_{tt}(z) T_{tt}(z') + O(g^2)$$

$$T_{tt} = \frac{1}{2}(\dot{\phi}^2 + \phi'^2)$$

Compute correlators at boundary of near-horizon (AdS2) region, then use free field equations to propagate them to null infinity

Correlator computations

Need to compute in-in expectation value:

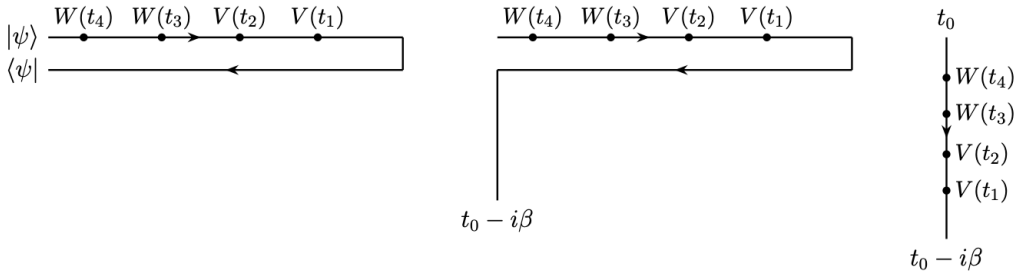


$|\psi_{in}\rangle = \text{Unruh vacuum}$

Insertions are at boundary of AdS2 region.

At low frequency Unruh and Hartle-Hawking vacua are equivalent near horizon

So in AdS2 region Unruh vacuum correlators become equivalent to thermal correlators, which can be computed in imaginary t and then continued back to Lorentzian:



Thermal correlators

For Euclidean AdS2 in coordinates $ds^2 = \frac{d\tau^2 + dz^2}{\sinh^2 z}\,, \quad \tau \cong \tau + 2\pi$

reduced scalar action is: $I[\phi] = \frac{1}{2}\int_0^{2\pi} d\tau \int_0^{\infty} dz \left(\dot{\phi}^2 + \phi'^2\right) - g\int_0^{2\pi} d\tau \int_0^{\infty} dz \int_0^{z} dz' \sinh^2 z' T_{\tau\tau}(z) T_{\tau\tau}(z') + O(g^2)$

$$T_{\tau\tau} = \frac{1}{2}\left(\dot{\phi}^2 - \phi'^2\right)$$

Compute boundary correlators from Witten diagrams as usual

Bulk-boundary propagator: $K(z,\tau;\tau_i) = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} e^{-|n|z - in(\tau-\tau_i)}$

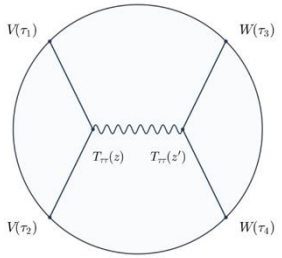
Bulk-bulk propagator: $G(\tau,z;\tau',z') = \frac{z_<}{2\pi} + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sinh(nz_<)\, e^{-nz_>}\cos(n(\tau-\tau'))$

Bilocal quartic vertex: $T_{\tau\tau}(z)T_{\tau\tau}(z')$

Bulk integrals are easy since only exponentials occur

Four-point function

$$G_4(\tau_i) = \langle V(\tau_1) V(\tau_2) W(\tau_3) W(\tau_4) \rangle =$$



$$+(z \leftrightarrow z')$$

Integrals are all trivial exponentials, giving $$G_4(\tau_i) = \frac{2g}{(2\pi)^3} \sum_{|k|>1} \frac{J_k^V(\tau_1,\tau_2) J_{-k}^W(\tau_3,\tau_4)}{k^2(k^2-1)}$$

$$J_k^V(\tau_1,\tau_2) = {\sum_{n_1,n_2}}' n_1 n_2 e^{in_1\tau_1 + in_2\tau_2} \delta_{n_1+n_2,k}, \qquad n_1 n_2 > 0$$

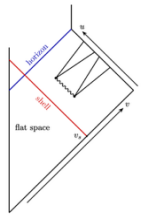
$$J_{-k}^W(\tau_3,\tau_4) = {\sum_{n_3,n_4}}' n_3 n_4 e^{in_3\tau_3 + in_4\tau_4} \delta_{n_3+n_4,-k}$$

Propagator comes from (quadratic part of) Schwarzian action: $$I_{\text{Sch}} = \frac{2\pi}{g} \sum_{|k|>1} k^2(k^2-1)\epsilon_k \epsilon_{-k}$$

$$G_4(\tau_i) = \int [D\epsilon] e^{-I_{\text{Sch}}} C^V(\tau_1,\tau_2) C^W(\tau_3,\tau_4) \qquad C(\tau_i,\tau_j) = -\frac{i}{\pi} \sum_{|k|>1} \text{sgn}(k) J_k(\tau_i,\tau_j) \epsilon_{-k}$$

Reproduces dressed bilocals coupled to Schwarzian [Maldacena,Stanford,Yang]

Four-point function on $\mathcal{I}^+$



$$\langle \partial_u V(u_1)\, \partial_u V(u_2)\, \partial_u W(u_3)\, \partial_u W(u_4) \rangle_c = \frac{(2\pi T_H)^4}{16} \frac{g}{4\pi^3} \frac{(-2 + 2\pi T_H u_{12} \coth(\pi T_H u_{12}))\,(-2 + 2\pi T_H u_{34} \coth(\pi T_H u_{34}))}{(2\sinh(\pi T_H u_{12}))^2\,(2\sinh(\pi T_H u_{34}))^2}.$$

Compare to free (disconnected) 4pt function $\dfrac{\pi T_H^2}{4\sinh^2(\pi T_H u_{12})} \dfrac{\pi T_H^2}{4\sinh^2(\pi T_H u_{34})}$

Comments

Connected part has relative factor of $g = \dfrac{G}{\pi r_0^3 T_H}$, becomes large as $T_H \to 0$ at fixed $T_H u_{ij}$

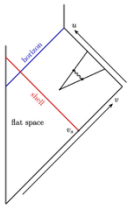
→ infrared effect

Fluctuations in canonical ensemble controlled by heat capacity $C_Q = \left.\dfrac{dM}{dT_H}\right|_Q \approx 4\pi^2 \sqrt{G} Q^3 T_H$

Indeed: $\langle \partial_u V(u_1)\, \partial_u V(u_2)\, \partial_u W(u_3)\, \partial_u W(u_4) \rangle_c = \frac{1}{C_Q} \left(T_H \frac{\partial}{\partial T_H} \langle \partial_u V(u_1)\, \partial_u V(u_2) \rangle \right) \left(T_H \frac{\partial}{\partial T_H} \langle \partial_u W(u_3)\, \partial_u W(u_4) \rangle \right)$

Two-point function on $\mathcal{I}^+$

2-point function



Other diagrams either vanish identically or by renorm. prescription setting scale free integrals to zero as in dim reg

$$\langle \partial_u \phi(u_1)\, \partial_u \phi(u_2)\rangle_{1-\text{loop}} = \frac{T_H^2 g}{8\sinh^2 x}\left[\frac{-2x^2-2\pi i x+1-\cosh(2x)+(i\pi+2x)\sinh(2x)}{4\sinh^2 x} - \left(-1+\frac{x}{\tanh x}\right)\left(-1+\frac{x+i\pi}{\tanh x}\right)\right]$$

$$x = \pi T_H u_{12}.$$

Correction to energy flux: $T_{uu}^{1-\text{ loop}} = 2\pi \lim_{u_1\to u_2} \langle \partial_u\phi(u_1)\,\partial_u\phi(u_2)\rangle_{1-\text{ loop}} = \frac{\pi g}{8} T_H^2 \sim g T_{uu}^{\text{tree}}$

Can be interpreted as correction to mass-energy relation

$$T_{uu}^{\text{tree}} = \frac{GE}{12 r_0^3} \qquad E = M - \frac{Q}{\sqrt{G}} = \frac{2\pi}{g} T_H + O(g^0)$$

1-loop correction

Write $T_{uu}^{1-\text{ loop}} = \frac{G\delta E}{12 r_0^3}$ 1-loop computation gives $\delta E = \frac{3}{2}\frac{\pi r_0^3}{G} g T_H^2 = \frac{3}{2} T_H$

From standard thermo relations $\ln Z(T_H) = \ln Z_{\text{cl}}(T_H) + \frac{3}{2}\ln T_H$

Reproduces 1-loop correction to Schwarzian path integral [Stanford,Witten]

Comments

- Hamiltonian form of action in AdS2 agrees on-shell with Schwarzian+bilocals -> all tree level correlators will agree. Loops more subtle
- OTOCs can be computed and usual Lyapunov growth obtained
- A different gauge choice allows for eternal two-sided black hole
- Bulk-boundary and bulk-bulk correlators can be computed
- In AdS3 case s-wave assumption can be dropped, and one gets a theory of bulk matter coupled to boundary gravitons
- Scattering