BMS particles

based on 2603.06297 w/ Yannick HERFRAY (IDP Tours)

2412.06002 w/ Yannick HERFRAY & Xavier BEKAERT (IDP Tours)

Laura DONNAY

Celestial Holography Simons Collaboration Workshop

Harvard, October 2026







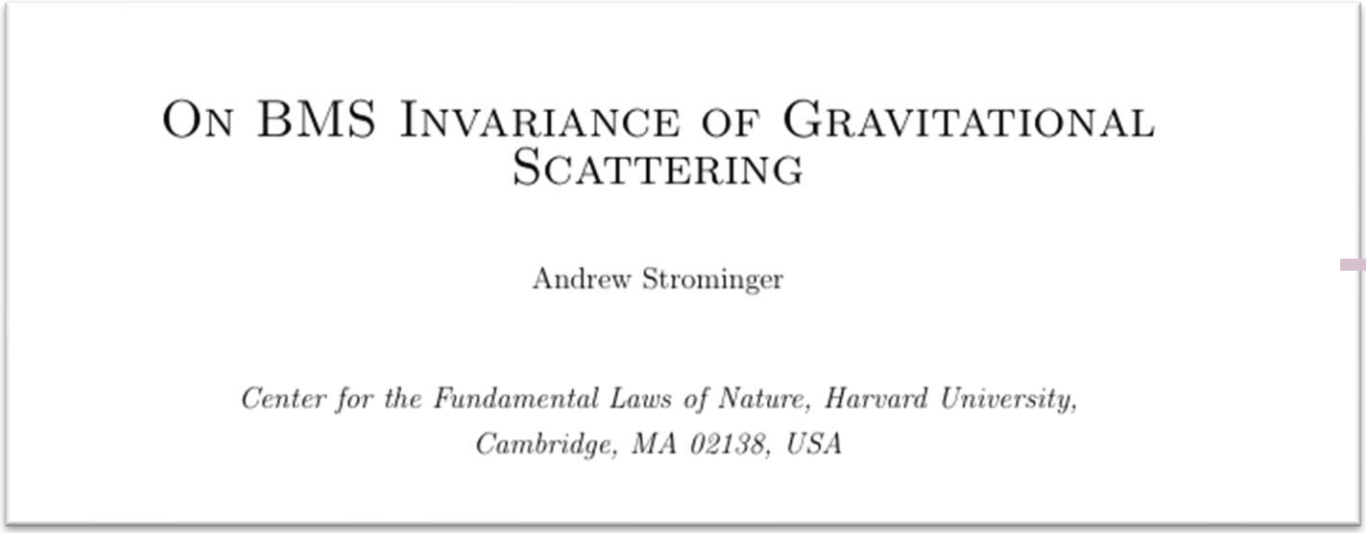
12 years ago… [Strominger ‘14]

On BMS Invariance of Gravitational Scattering

Andrew Strominger

Center for the Fundamental Laws of Nature, Harvard University,
Cambridge, MA 02138, USA

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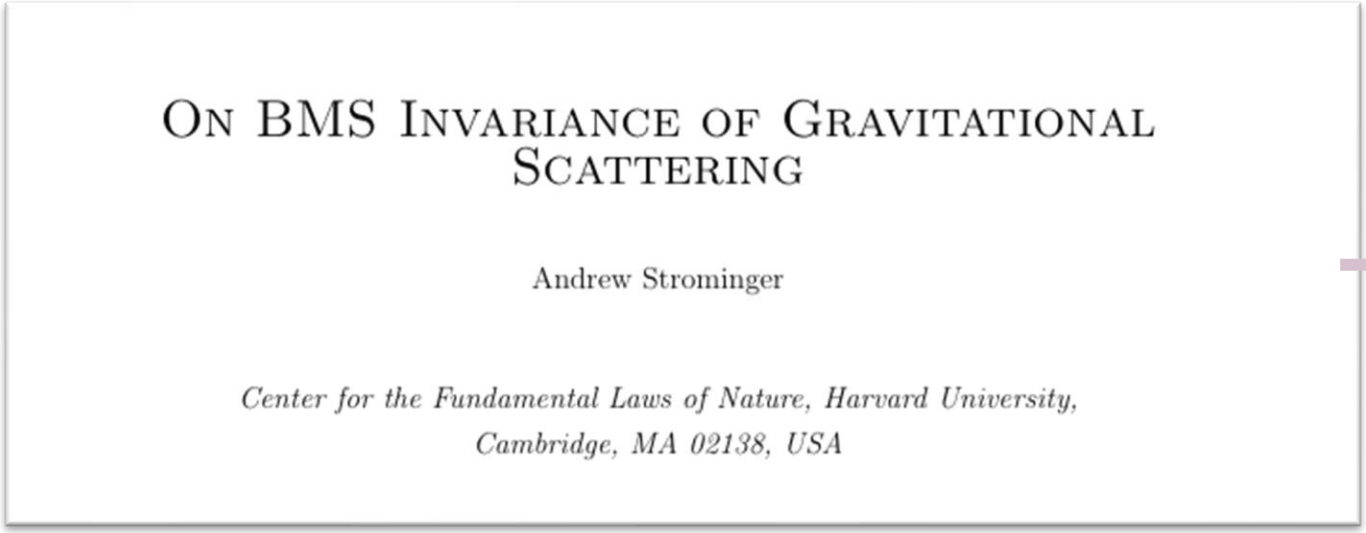
ON BMS INVARIANCE OF GRAVITATIONAL SCATTERING

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The realization that
the **S-matrix** is **BMS invariant**
was foundational for
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12 years ago… [Strominger '14]



ON BMS INVARIANCE OF GRAVITATIONAL SCATTERING

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→ The realization that the **S-matrix** is **BMS invariant** was foundational for celestial holography

This talk:

Work out the **representation theory** of **BMS symmetries** (and their QED analog)

↑ rigid framework for flat space holography

AdS / CFT correspondence

is a statement about the equivalence of
2 quantum theories

→ $\exists$ an isomorphism between their Hilbert spaces

and their algebras of physical observables are equivalent

Symmetries play a fundamental role: fields in AdS and CFT operators realize the **same representations** of the conformal algebra

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In flat space holography

Unitary **I**rreducible **R**epresentations of the boundary symmetry group??

$\equiv$ **BMS particle***

* See [Barnich, Oblak '14 '15] for UIR of BMS_3

What Hilbert space are we trying to map?

Flat space holography

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Tightly connected to the issue of **infrared divergences**

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[Chung '65][Kibble '68][Dollard '71][Kulish, Faddeev '70] (see [Ashtekar '84] for gravity)

Faddeev-Kulish (FK) dressed states $\in$ *enlarged* Hilbert space $\mathcal{H}_{\rm FK}$

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→ *Modern revisit:*

IR divergences arise, for conventional states, as a necessity to **respect BMS conservation laws.**

[Kapec, Perry, Raclariu, Strominger '17][Akhoury, Choi '17]

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↓

Pushing this idea further:

states of the S-matrix should be BMS states!

Outline

based on

- BMS particles
  - hard UIRs
  - generic representations

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- Relation with infrared physics

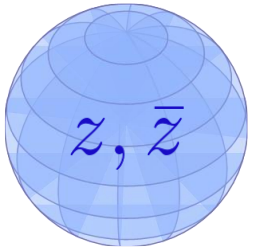
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- Outlook

Asymptotic symmetry group of gravity

$$\mathrm{BMS}_4 \simeq \mathrm{SO}(3,1) \ltimes \mathcal{E}[1](S^2)$$

Lorentz acts on the celestial sphere



$S^2 \cong \mathbb{C} \cup \{\infty\}$

as $z \to z' = \dfrac{az+b}{cz+d}$ $SL(2,\mathbb{C})$

$ad - bc = 1$

supertranslations

$\mathcal{T}(z, \bar{z})$

= conformal densities of weight–1

$(h, \bar{h}) = (-\frac{1}{2}, -\frac{1}{2})$

$\mathcal{E}[w]$: conformal densities of weights $(h, \bar{h}) = (-\frac{w}{2}, -\frac{w}{2})$

Important reminder

- The Poincaré group sits inside BMS:

$$ISO(3,1) \simeq SO(3,1) \ltimes \mathbb{R}^{3,1} \subset BMS_4 \simeq SO(3,1) \ltimes \mathcal{E}[1](S^2)$$

However, the inclusion is not unique

→ Infinitely many non-equivalent Poincaré groups inside BMS

[Ashtekar, 1984]

choice of $ISO(3,1) \subset BMS_4$ = choice of ``gravity vacuum''

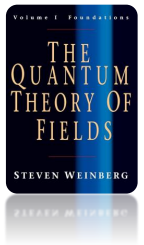
- But, there is a canonical inclusion of translations into supertranslations

$$q_\mu \left| \begin{array}{ccc} \mathbb{R}^{3,1} & \hookrightarrow & \mathcal{E}[1] \\ T^\mu & \mapsto & \mathcal{T}(z,\bar{z}) = T^\mu q_\mu(z,\bar{z}) \end{array} \right.$$

$$q^\mu(z,\bar{z}) = (1+|z|^2, z+\bar{z}, -i(z-\bar{z}), 1-|z|^2)$$

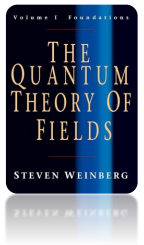
BMS particles

- classification -



Poincaré

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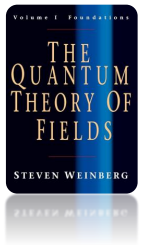


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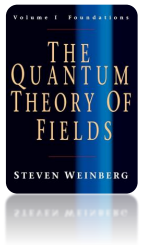
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In practice, fix a reference momentum k_μ
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$$\ell_k = \{\Lambda \in SO(3,1) \mid \Lambda k = k\}$$

Wigner's classification

little group	ℓ_k	$SU(2)$	$ISO(2)$	$SL(2,\mathbb{R})$
mass-shell	$\mathcal{O}_k$	H^3	$\mathbb{R} \times S^2$	dS_3



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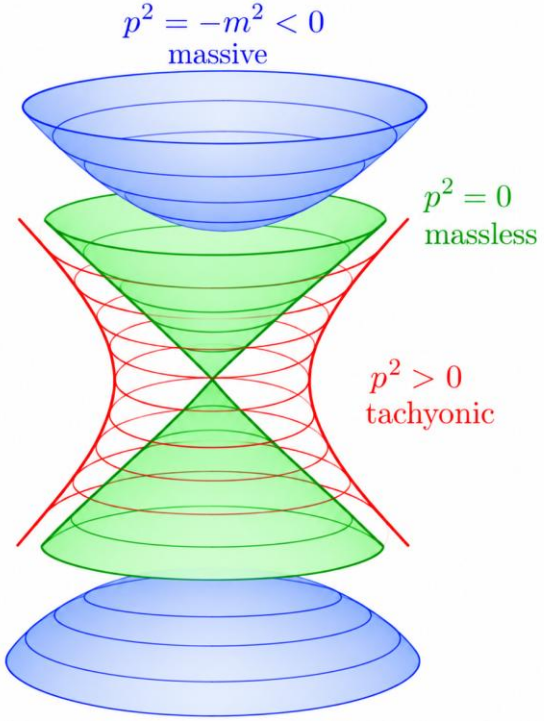
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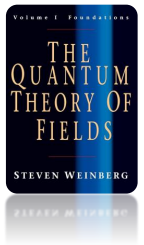
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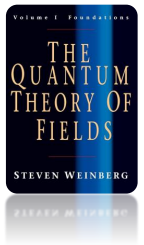
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= wavefunction with support on $\mathcal{O}_k$

+ spin (UIR of little group)



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See [Mackey '68] and [Piard '77] for induced representations of groups of the form $SO(3,1) \ltimes A$

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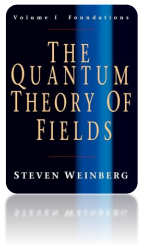
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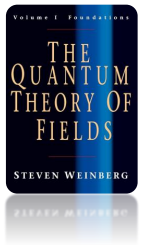
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Task: find them all

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Seminal papers

[1] V. Cantoni, *A Class of Representations of the Generalized Bondi—Metzner Group*, **J. Math. Phys.** 7 (1966), 1361–1364.

[2] V. Cantoni, *Reduction of Some Representations of the Generalized Bondi-Metzner Group*, **J. Math. Phys.** 8 (1967), 1700–1706.

[3] P. J. M. McCarthy, *Asymptotically flat space-times and elementary particles*, **Phys. Rev. Lett.** 29 (1972) 817–819.

[4] P. J. McCarthy, *Representations of the Bondi-Metzner-Sachs Group. I. Determination of the Representations*, **Proc. R. Soc. Lond. A** 330 (1972), no. 1583, 517–535.

[5] P. J. McCarthy, *Representations of the Bondi-Metzner-Sachs Group. II. Properties and Classification of the Representations*, **Proc. R. Soc. Lond. A** 333 (1973), no. 1594, 317–336.

[6] P. J. McCarthy and M. Crampin, *Representations of the Bondi-Metzner-Sachs Group. III. Poincare Spin Multiplicities and Irreducibility*, **Proc. R. Soc. Lond. A** 335 (1973), no. 1602, 301–311.

[7] L. Girardello and G. Parravicini, *Continuous spins in the Bondi-Metzner-Sachs group of asymptotic symmetry in general relativity*, **Phys. Rev. Lett.** 32 (1974), 565–568.

[8] P. J. McCarthy, *The Bondi-Metzner-Sachs Group in the Nuclear Topology*, **Proc. R. Soc. Lond. A** 343 (1975), no. 1635, 489–523.

[9] M. Crampin and P. J. McCarthy, *Representations of the Bondi-Metzner-Sachs Group. IV. Cantoni Representations are Induced*, **Proc. R. Soc. Lond. A** 351 (1976), no. 1664, 55–70.

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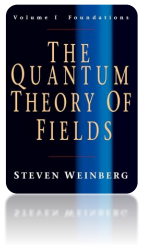
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Note: confusions and errors for the massless case



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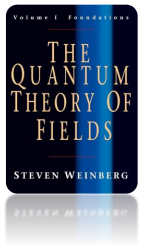
[McCarthy '70s] : there are **11** possible **little groups**

$\ell_\mathcal{K}$ with mass-shell of dimensions

$$Dim\left(\mathcal{O}_\mathcal{K}\right) \in \{0,3,4,5,6\}$$

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BMS particle
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	Poincaré little group ℓ_p	**Orbit** (mass shell) $\frac{SL(2,\mathbb{C})}{\ell_p}$	**BMS little group** $\ell_{\mathcal{P}}$	**Extra d.o.f.** (Fibre of BMS shell) $\frac{\ell_p}{\ell_{\mathcal{P}}}$	**BMS shell dim.**
Massless momentum	$ISO(2)$	$\mathbb{R}^+ \times S^2$	$ISO(2) = \ell_p$	$\{e\} =$ trivial	3
			$\mathbb{R}^2$	S^1	4
			$\mathbb{R}$	$S^1 \times \mathbb{R}$	5
			$U(1)$	$\mathbb{R}^2$	5
			$\{e\} =$ trivial	$ISO(2) = S^1 \times \mathbb{R}^2$	6
Massive momentum	$SU(2)$	$\mathbb{H}^3$	$SU(2) = \ell_p$	$\{e\} =$ trivial	3
			$U(1)$	S^2	5
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taken from [LD, Herfray '26]

Questions we would like to answer:

What do BMS states look like?
Where is the physics?
How do BMS particles differ from Poincaré particles? Extra IR degrees of freedom?

hard BMS UIR

	Poincaré little group ℓ_p	Orbit (mass shell) $\frac{SL(2,\mathbb{C})}{\ell_p}$	BMS little group $\ell_{\mathcal{P}}$	Extra d.o.f. (Fibre of BMS shell) $\frac{\ell_p}{\ell_{\mathcal{P}}}$	BMS shell dim.
Massless momentum	$ISO(2)$	$\mathbb{R}^+ \times S^2$	$ISO(2) = \ell_p$	$\{e\}$ = trivial	3
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Poincaré generators:

$$\hat{P}^\mu = \int \frac{d^3p}{(2\pi)^3\, 2p^0}\, p^\mu \hat{b}^\dagger(\vec{p})\hat{b}(\vec{p}) \qquad \hat{J}^{\mu\nu} = i\int \frac{d^3p}{(2\pi)^3\, 2p^0} \left(p^\mu \frac{\partial}{\partial p_\nu} - p^\nu \frac{\partial}{\partial p_\mu}\right) \hat{b}^\dagger(\vec{p})\hat{b}(\vec{p})$$

Poincaré algebra:

$$\left[\hat{P}^\mu, \hat{P}^\nu\right] = 0$$
$$\left[\hat{J}^{\mu\nu}, \hat{P}^\rho\right] = i\eta^{\nu\rho}\hat{P}^\mu - i\eta^{\mu\rho}\hat{P}^\nu$$
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Hard BMS UIRs

Bekaert – LD – Herfray

Each momentum can be associated to a hard supermomentum via

$$p^\mu = \int d^2z\, P(z,\bar z) q^\mu(z,\bar z) \qquad q^\mu(z,\bar z) = (1+|z|^2, z+\bar z, -i(z-\bar z), 1-|z|^2)$$

$$P(z,\bar z) = \begin{cases} \omega\, \delta^{(2)}(z-\zeta) & \text{if } p^2 = 0,\ p_\mu = \omega q_\mu(\zeta,\bar\zeta), \\ -\dfrac{m^4}{2\pi\,(q(z,\bar z)\cdot p)^3} & \text{if } p^2 < 0\,. \end{cases}$$

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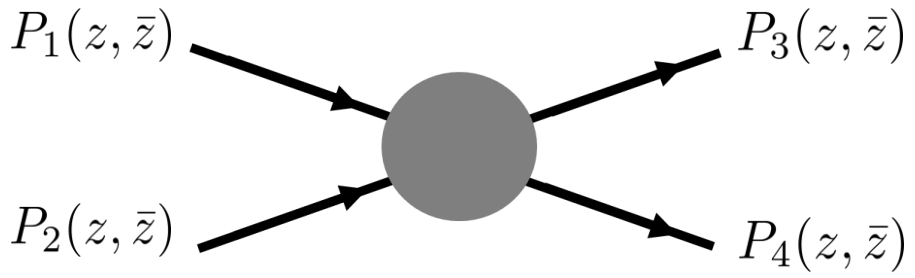
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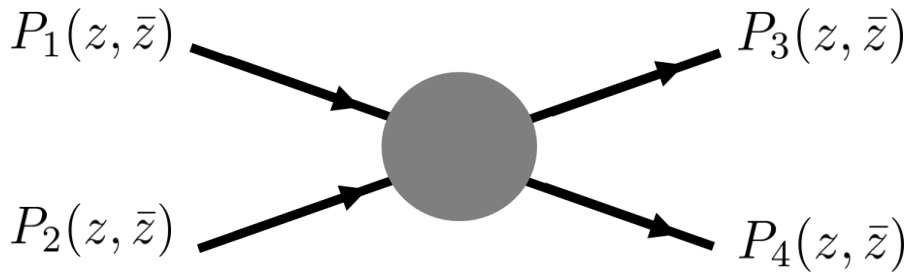
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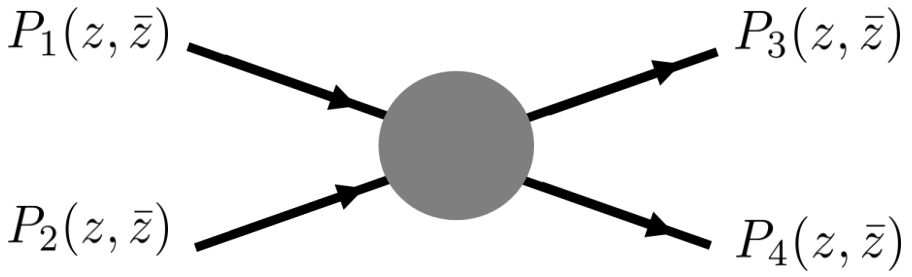
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obstruction to supermomentum conservation

(generic) BMS particles

Supermomenta decomposition

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$$\mathcal{P}(z,\bar{z}) = \partial_z^2 \partial_{\bar{z}}^2 \mathcal{N} \quad + P(z,\bar{z})$$

[Bekaert, LD, Herfray '24]
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$$\mathcal{N}(z,\bar{z}) \in \mathcal{E}[1]/\mathbb{R}^{3,1}$$

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This decomposition is unique. It is Lorentz invariant and nonlinear.

BMS wavefunctions

[Bekaert, LD, Herfray '24]

BMS wavefunctions

$$|\Psi\rangle = \int_{\mathcal{O}_{\mathcal{P}}} d\mu(\mathcal{P})\, \Psi(\mathcal{P})|\mathcal{P}\rangle$$

cf. usual wavefunctions

$$|\Psi\rangle = \int_{\mathcal{O}_p} d\mu(p)\Psi(p)|p\rangle$$

using $\mathcal{P}(z,\bar{z}) = \partial_z^2\partial_{\bar{z}}^2\mathcal{N} + P(z,\bar{z})$

$$|\Psi\rangle = \int \omega d\omega d^2z \int \mathcal{D}\mathcal{N}\ \Psi(\omega, z; \partial^2\mathcal{N})|\omega, z; \partial^2\mathcal{N}\rangle$$

finite-dimensional integrals*

wavefunctions with support
on a given orbit of the Lorentz group

* see [Bekaert, Herfray '25] for explicit examples

In QED

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hard part soft part

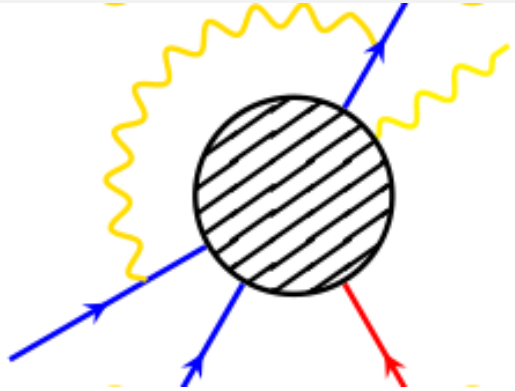
IR physics

Supermomenta and IR divergences (in QED and gravity)

- Obstruction of supermomentum conservation:

$$\mathcal{S}(z,\bar{z}) = \frac{1}{2\pi}\sum_i \eta_i q_i \ln|p_i \cdot q(z,\bar{z})|$$

is precisely Weinberg's **soft factor**



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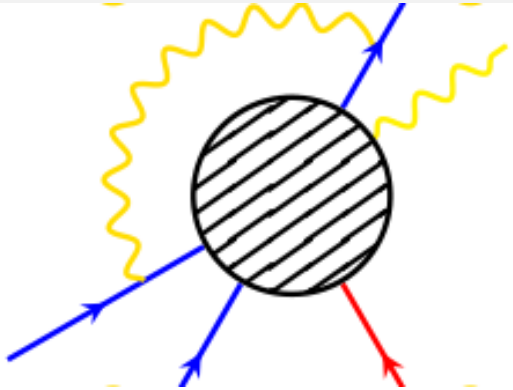
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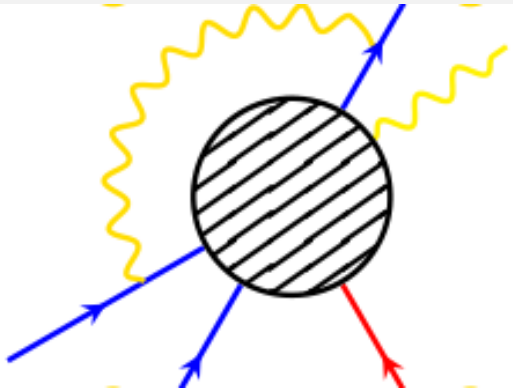
- Its **Lorentz-invariant norm** (e.g. [Gelfand, Graev, Vilenkin '66])

$$\|\mathcal{S}\|^2 = \int d^2z\, \partial_z \mathcal{S} \partial_{\bar{z}} \mathcal{S}$$



[LD, Herfray '26]

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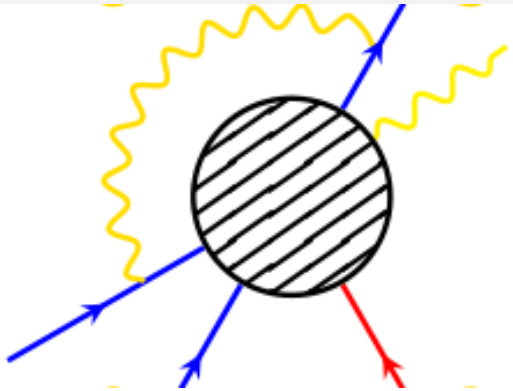
$$= \sum_{i,j} \eta_i\eta_j \frac{q_i q_j}{(2\pi)^2} \int d^2z\, \partial_z\Big(\ln|p_i\cdot q(z,\bar{z})|\Big)\,\partial_{\bar{z}}\Big(\ln|p_j\cdot q(z,\bar{z})|\Big)$$

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see also [Agrawal, Nguyen '25]

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→ gives the **exponentiation formula** for **virtual divergences**

$$\mathcal{A} = e^{-\mathcal{W}} A_{\text{IR-finite}} \qquad \mathcal{W} = \log\left(\frac{\Lambda}{\lambda}\right)\frac{1}{16\pi^2}\sum_{i,j}\frac{\eta_i\eta_j\, q_i q_j}{\beta_{ij}}\ln\frac{1+\beta_{ij}}{1-\beta_{ij}}$$

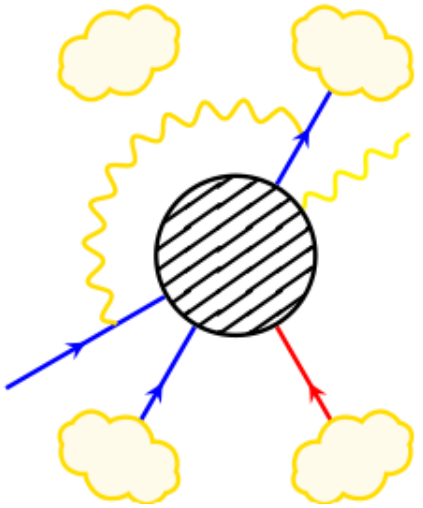
[Yennie, Frautschi, Suura '61][Weinberg '65]

UIR vs dressed states

Comparison with Faddeev-Kulish dressed states

FK dressed states

$$|\vec{p}\rangle_{\mathrm{FK}} = e^{\hat{R}} |\vec{p}\rangle$$



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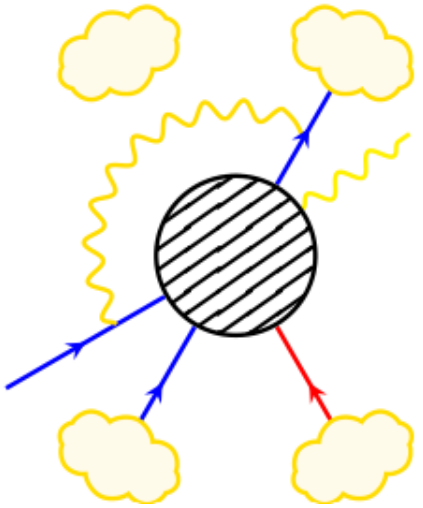
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photon momentum

$\psi(k,p) = 1$ in a neighborhood of $\varpi = 0$



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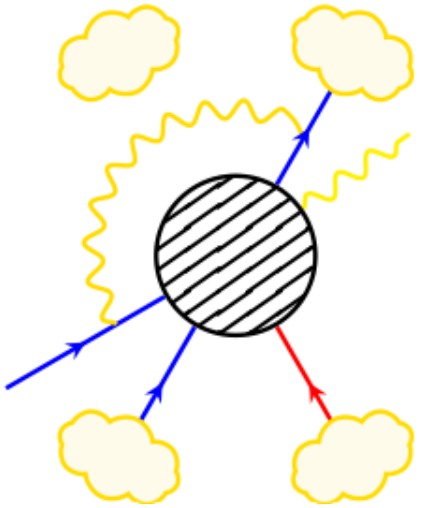
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c-vector should satisfy $c \cdot k = 1$ (gauge invariance)

Comparison with Faddeev-Kulish dressed states

FK dressed states

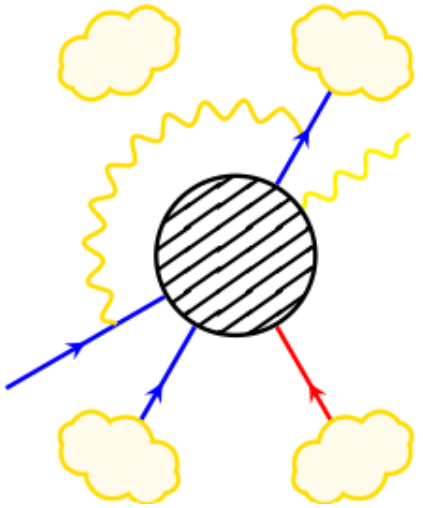
$$|\vec{p}\rangle_{\rm FK} = e^{\hat{R}}\,|\vec{p}\rangle$$

$$\hat{R} = -e\int \frac{d^3k}{(2\pi)^3 2k^0}\left[f^\mu(k,p)\hat{a}^\dagger_\mu(k) - f^{*\mu}(k,p)\hat{a}_\mu(k)\right]$$

$$f^\mu(k,p) = q_e\left(\frac{p^\mu}{p\cdot k} - c^\mu\right)\psi(k,p)$$

$k^\mu = \varpi q^\mu(z,\bar{z})$
photon momentum

$\psi(k,p) = 1$ in a neighborhood of $\varpi = 0$



- c-vector should satisfy $c\cdot k = 1$ (gauge invariance)

  [Kulish, Faddeev '70] took $c\cdot\varepsilon = 0$ → $F^\mu(k,p) = q_e\frac{p^\mu}{p\cdot k}\psi(k,p)$ → IR divergences cancel [Chung '65]

- $|\vec{p}\rangle_{\rm FK}$ is an eigenstate of the *soft part* of the supermomentum [Gabai, Sever '16] [Kapec, Perry, Raclariu, Strominger '17]

  but it is not a momentum (nor supermomentum) eigenstate

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How to turn FK into a supermomentum eigenstate:

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Take the dressing factor

$$f^\mu = q_e\left(\frac{p^\mu}{p\cdot k} - \frac{\hat{c}^\mu}{\varpi}\right)\psi_\epsilon(\varpi) \qquad \text{with} \qquad \psi_\epsilon(\varpi) = \begin{cases} 1 & \text{if} \quad 0 \le \varpi < \epsilon \\ 0 & \text{otherwise} \end{cases}$$

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$$= \frac{q_e m^2}{4\pi\,(q(z,\bar{z})\cdot p)^2} - \frac{q_e}{2\pi}\partial_z\partial_{\bar{z}}\ln|p\cdot q| + \frac{q_e}{2\pi}\,\partial_{\bar{z}}\,(\hat{c}\cdot\partial_z q)$$

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$$= q_e\,\delta^{(2)}(z-\infty,\bar{z}-\infty) + \frac{q_e}{2\pi}\,\partial_{\bar{z}}\,(\hat{c}\cdot\partial_z q)$$

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Key remarks:

- all c_i 's are equal → *conservation of electric charge* $\sum_{i\in \text{in}} q_i = \sum_{i\in \text{out}} q_i$ ⟺ *conservation of supermomentum* $\sum_i \mathcal{P}_i = 0$ ⟺ *IR finiteness*

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  ⟺ *IR finiteness*

  $$\Delta c^\mu := \sum_{i\in\text{in}} q_i \hat{c}_i^\mu - \sum_{i\in\text{out}} q_i \hat{c}_i^\mu$$

  [LD, Herfray '26] (see also [Gabai, Sever '16])

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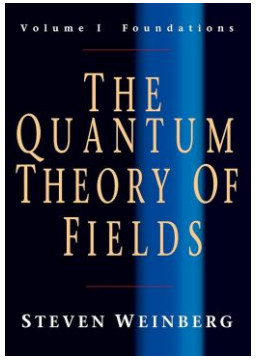
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[Bekaert, LD, Herfray '24]
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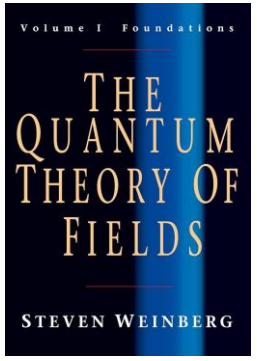
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Thank you!