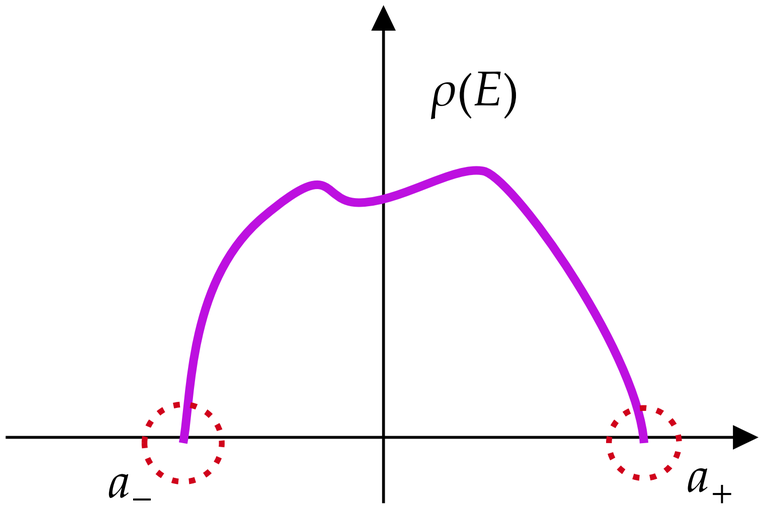
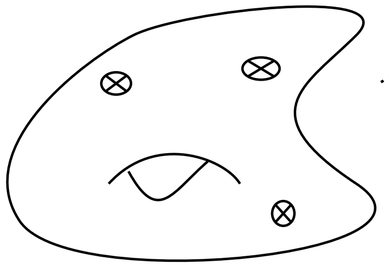
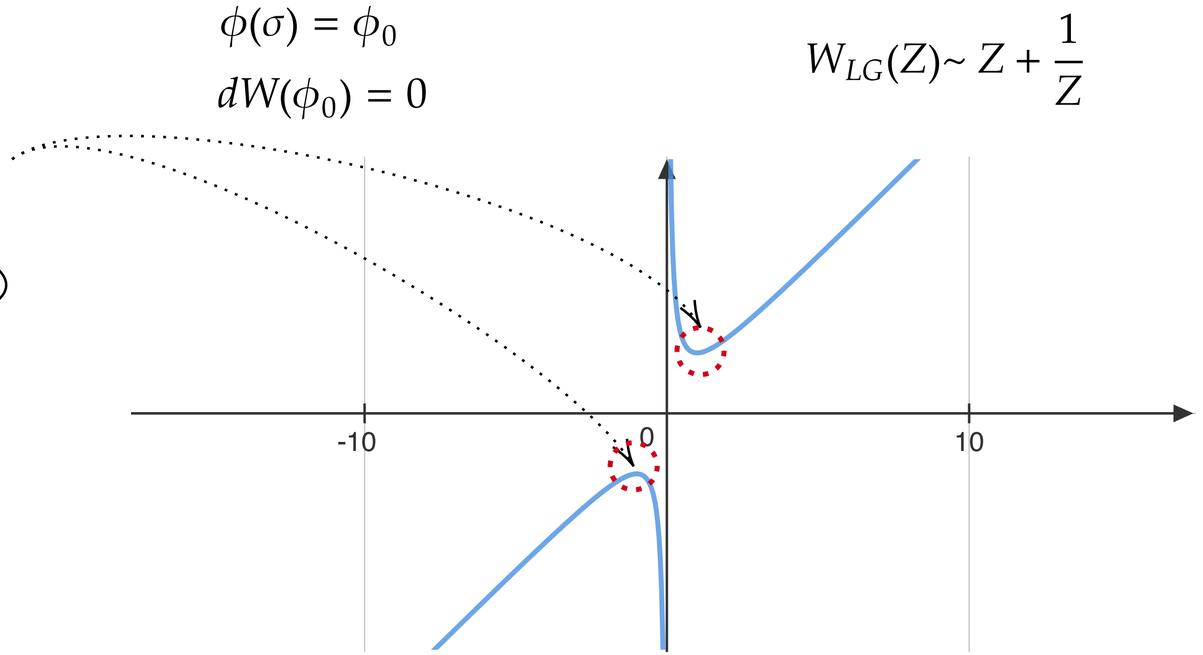
Finite-dimensional Holography

(to be followed up by Rajesh's talk tomorrow)







arXiv: 2604.03126 + WIP , w/ A. Giacchetto (ETH) & R. Gopakumar (ICTS)

Edward Mazenc (ETH Zürich) - Oct. 29, 2026
Simons Celestial Holography Workshop, Harvard

The Dream

Can we ``derive" holography?
(e.g. AdS/CFT, Chiral-Twisted Holography,…)

The Dream

Can we ``derive" holography?
(e.g. AdS/CFT, Chiral-Twisted Holography,…)

Can we determine the bulk dual to a given boundary theory?

The Dream

Today:

do this for the simplest large N theory

- the 1-matrix model -

The Dream

Today:

do this for the simplest large N theory

- the 1-matrix model -

→ no CFT, SUSY, AdS, brane construction as guiding principles

The Big Picture(s)

A cartoon summary of this talk

Matrix Model Holography

Matrix integral

$$\int dM_{N\times N}\, e^{-N \operatorname{Tr} V(M)}$$



Boundary

0d

Matrix Model Holography

Matrix integral

$$\int dM_{N\times N}\, e^{-N\,\mathrm{Tr}\,V(M)}$$



Boundary

0d

Finite dimensional integral over N^2 variables!

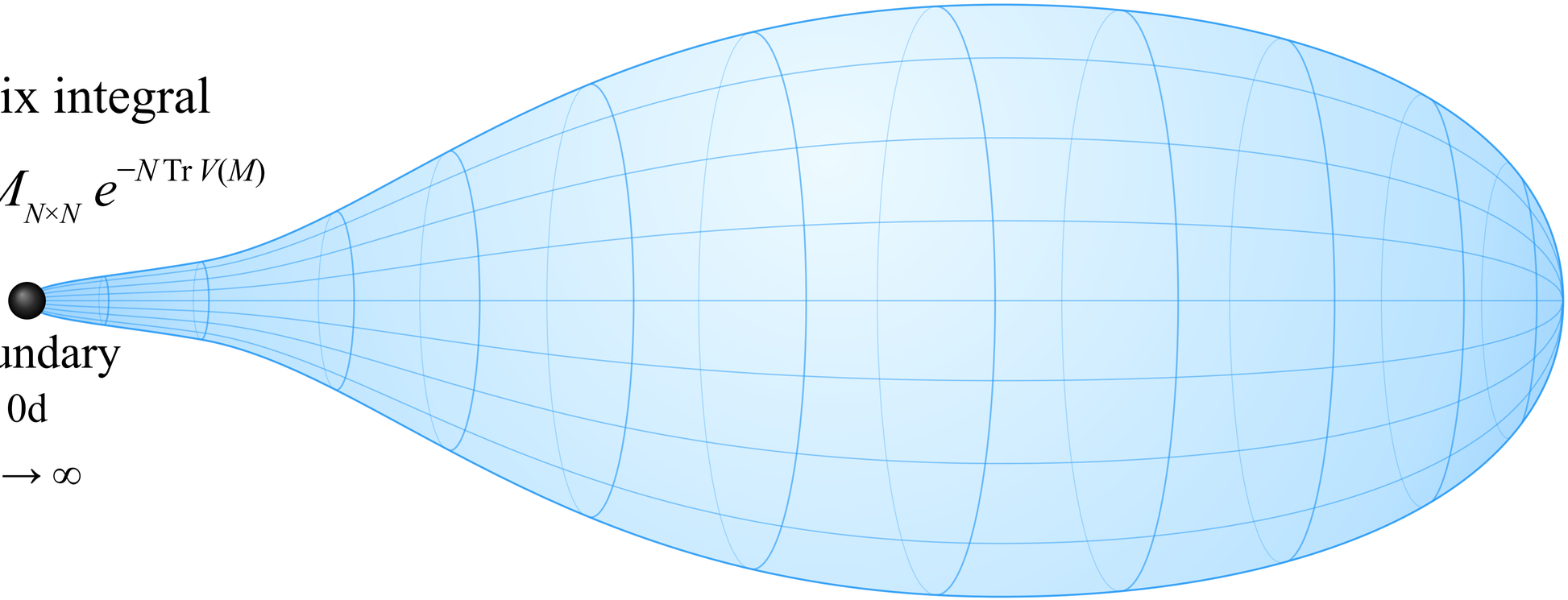
Matrix Model Holography

Bulk 2d

Emergent geometry at large N

Matrix integral

$$\int dM_{N\times N}\, e^{-N \operatorname{Tr} V(M)}$$



Boundary

0d

$x \longrightarrow \infty$

Spectral curve

$$y^2 - V'(x)y + P(x) = 0$$

['t Hooft ('74); Brézin, Itzykson, Parisi, Zuber ('78); Dijkgraaf, Vafa ('02)]

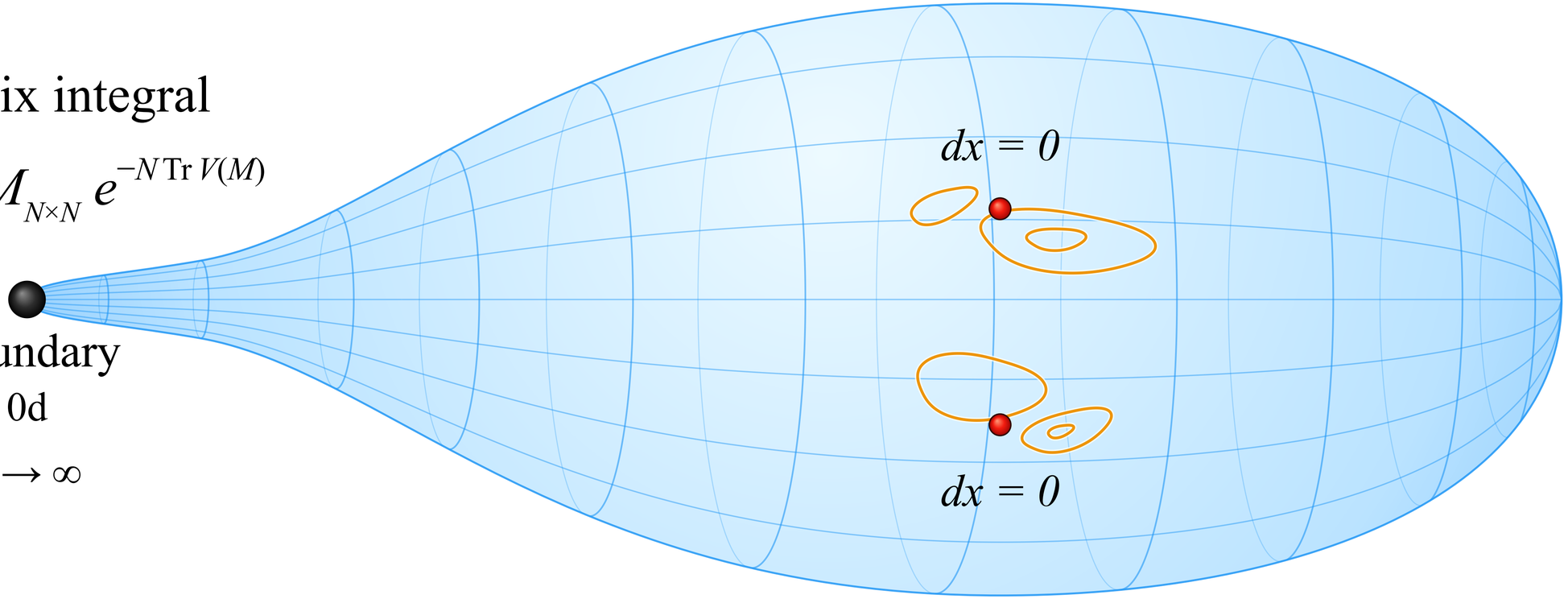
Matrix Model Holography

Bulk 2d

Target space of a string theory [*c.f.* Dijkgraaf, Vafa ('02)]

Matrix integral

$$\int dM_{N\times N}\, e^{-N \operatorname{Tr} V(M)}$$



Boundary

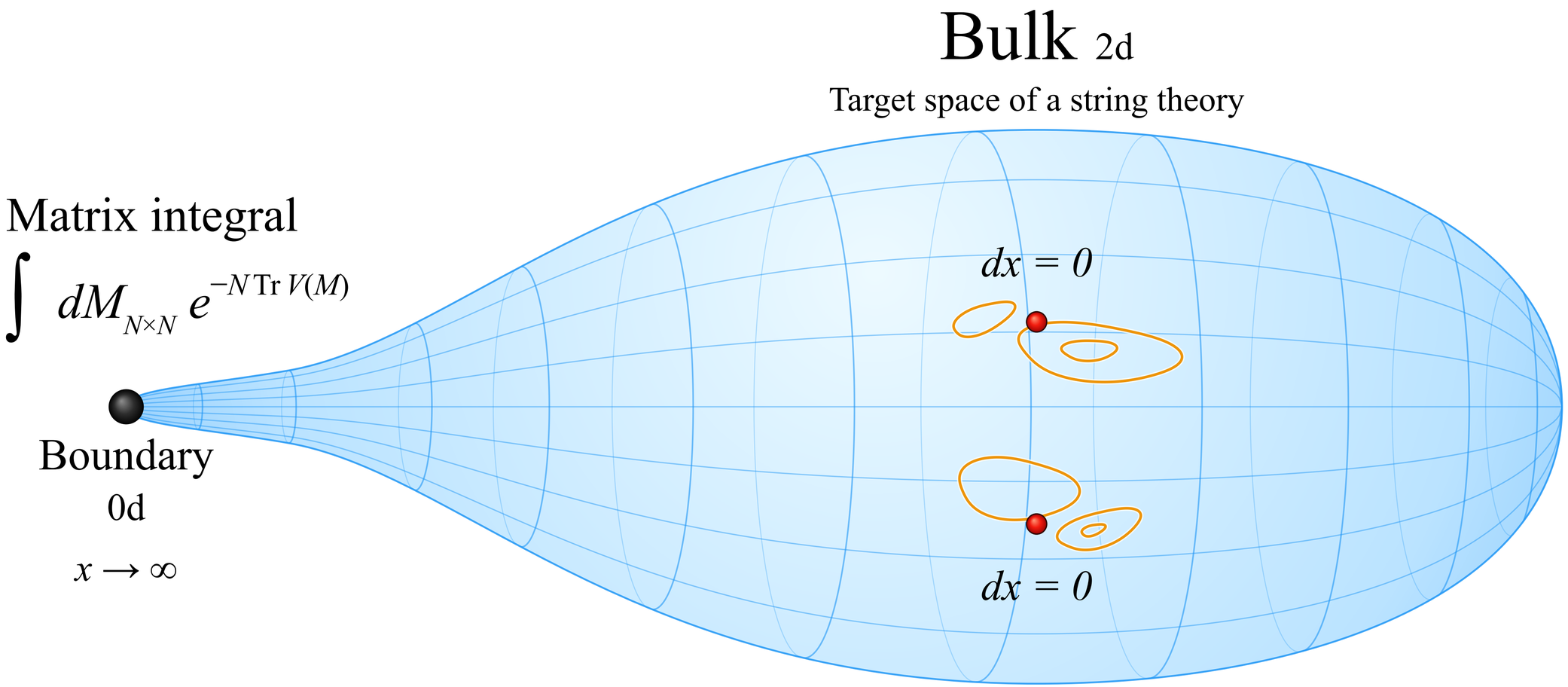
0d

$x \to \infty$

Spectral curve

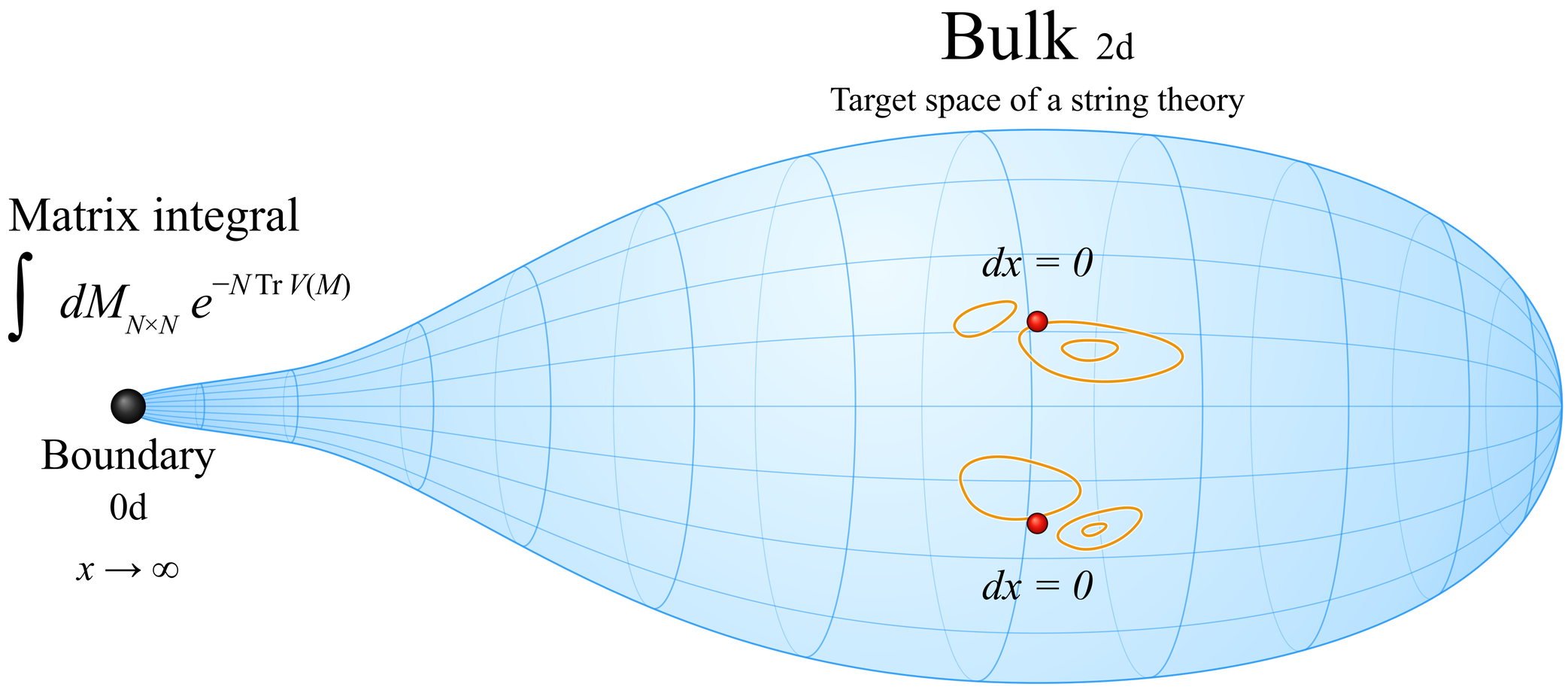
$$y^2 - V'(x)y + P(x) = 0$$

Matrix Model Holography



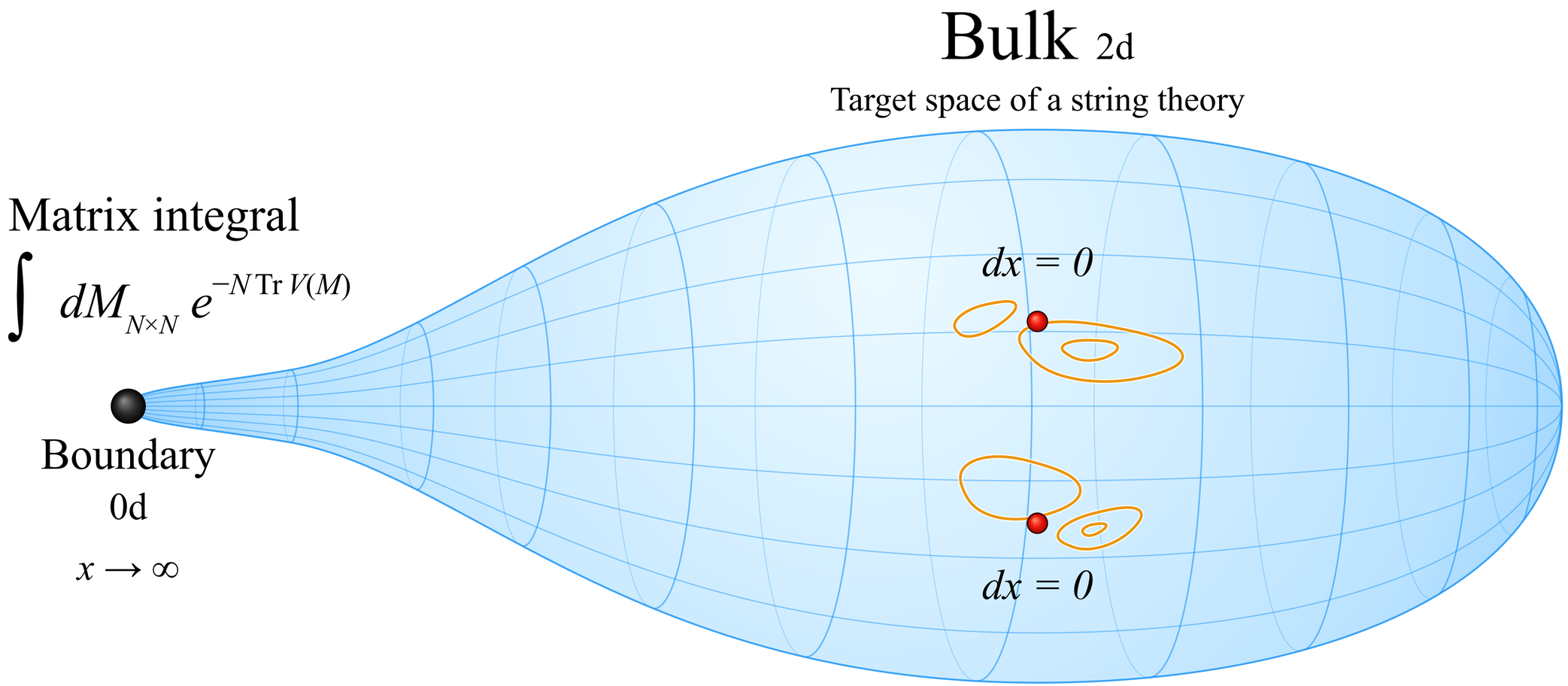
What string theory?

Matrix Model Holography



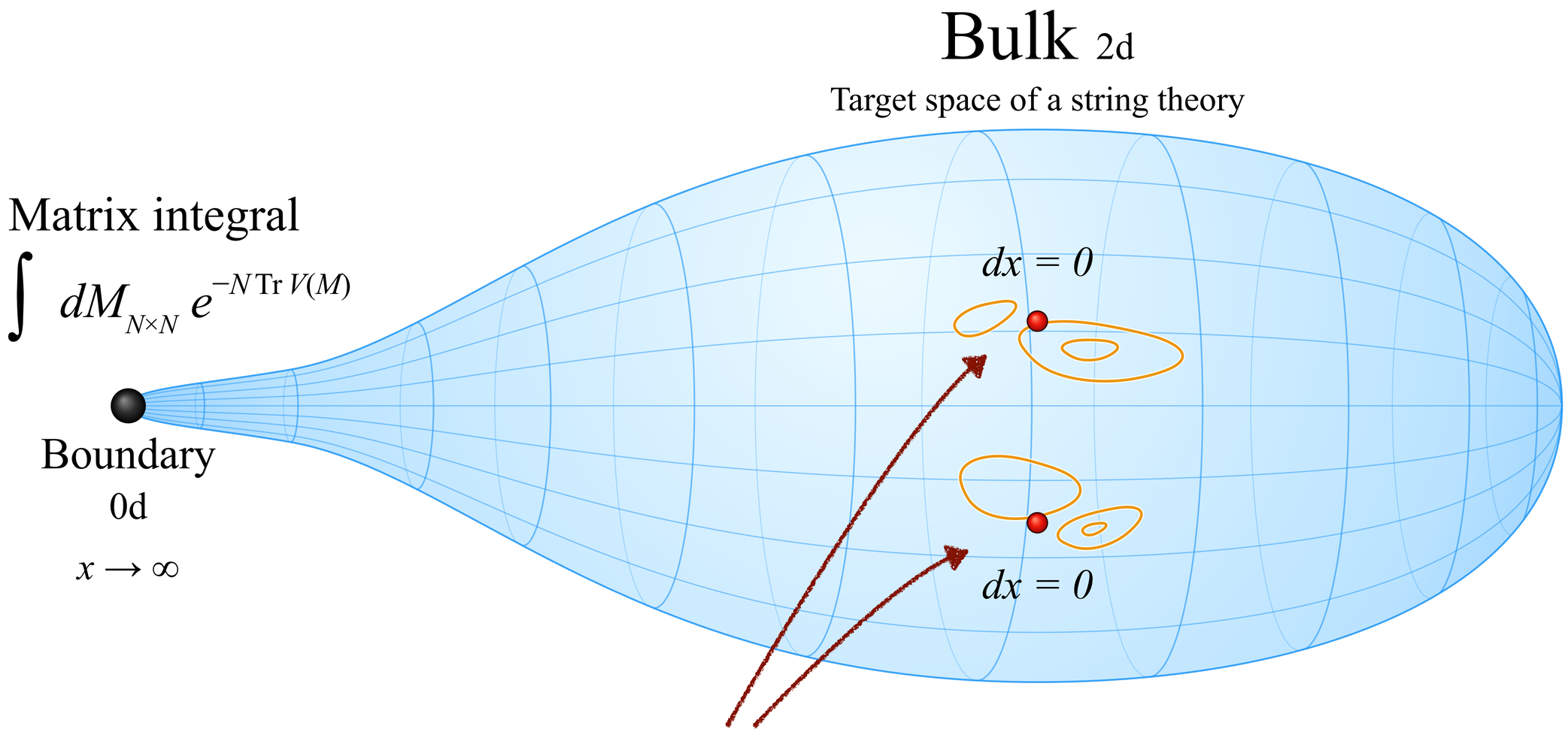
Our finding: ***B-twisted LG model coupled to 2d top gravity***

Matrix Model Holography



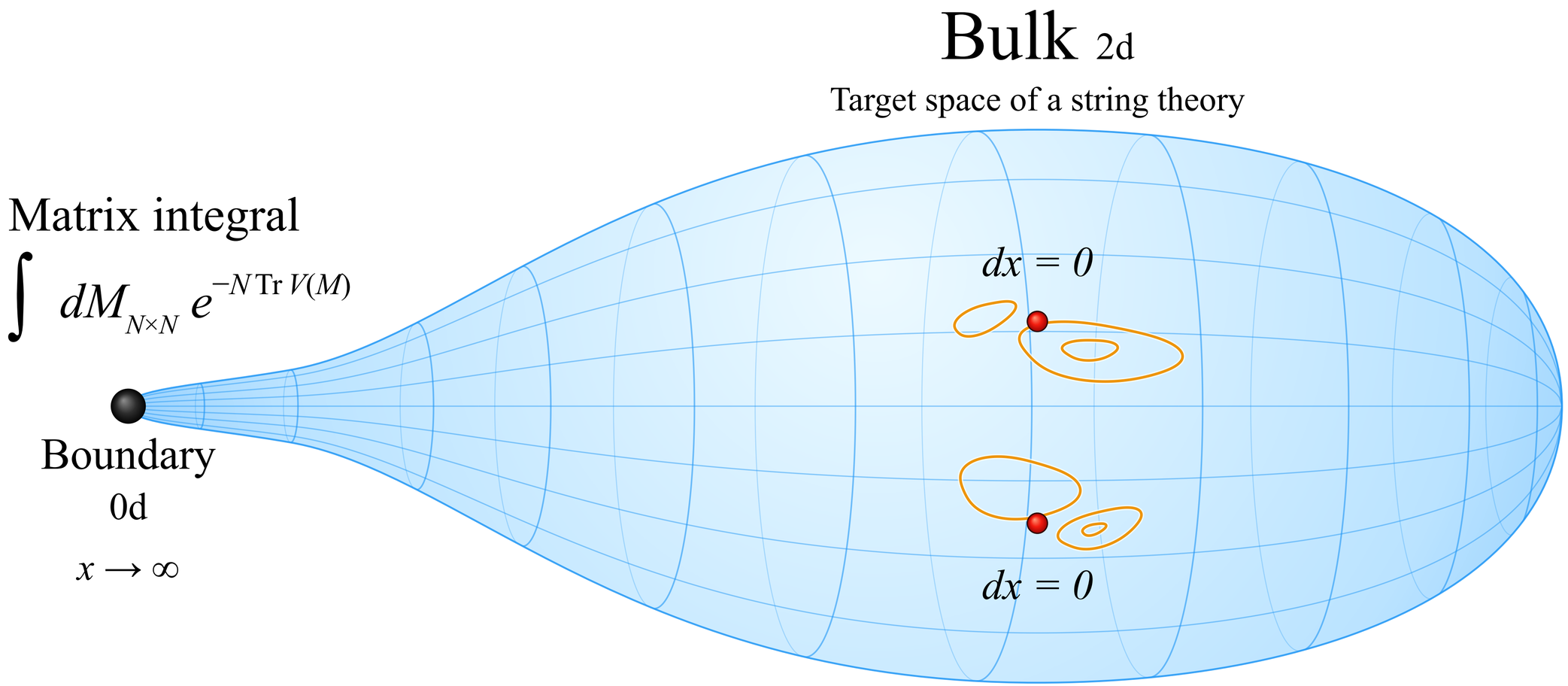
Dictionary: $W_{LG} \leftrightarrow x$ $\Omega = dy$

Matrix Model Holography



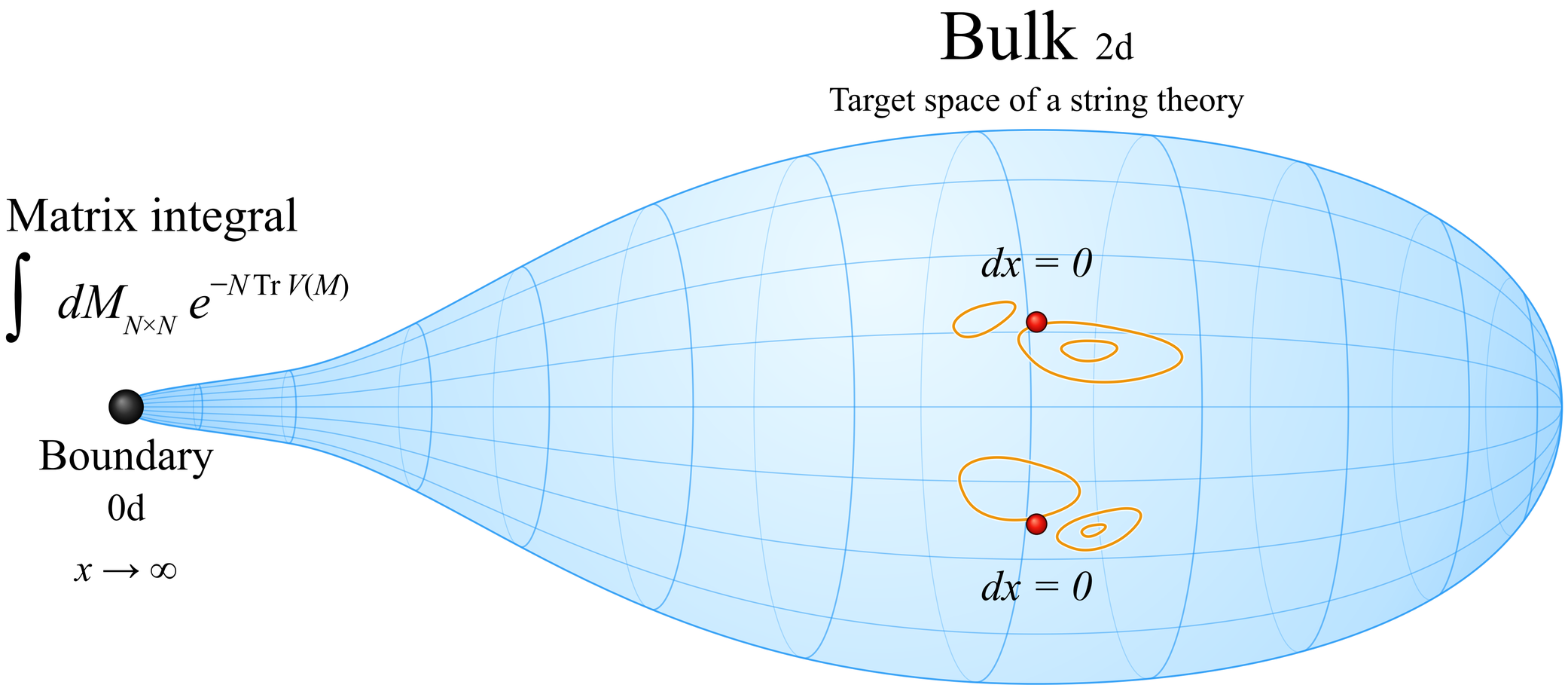
Critical points of superpotential W_{LG}

Matrix Model Holography



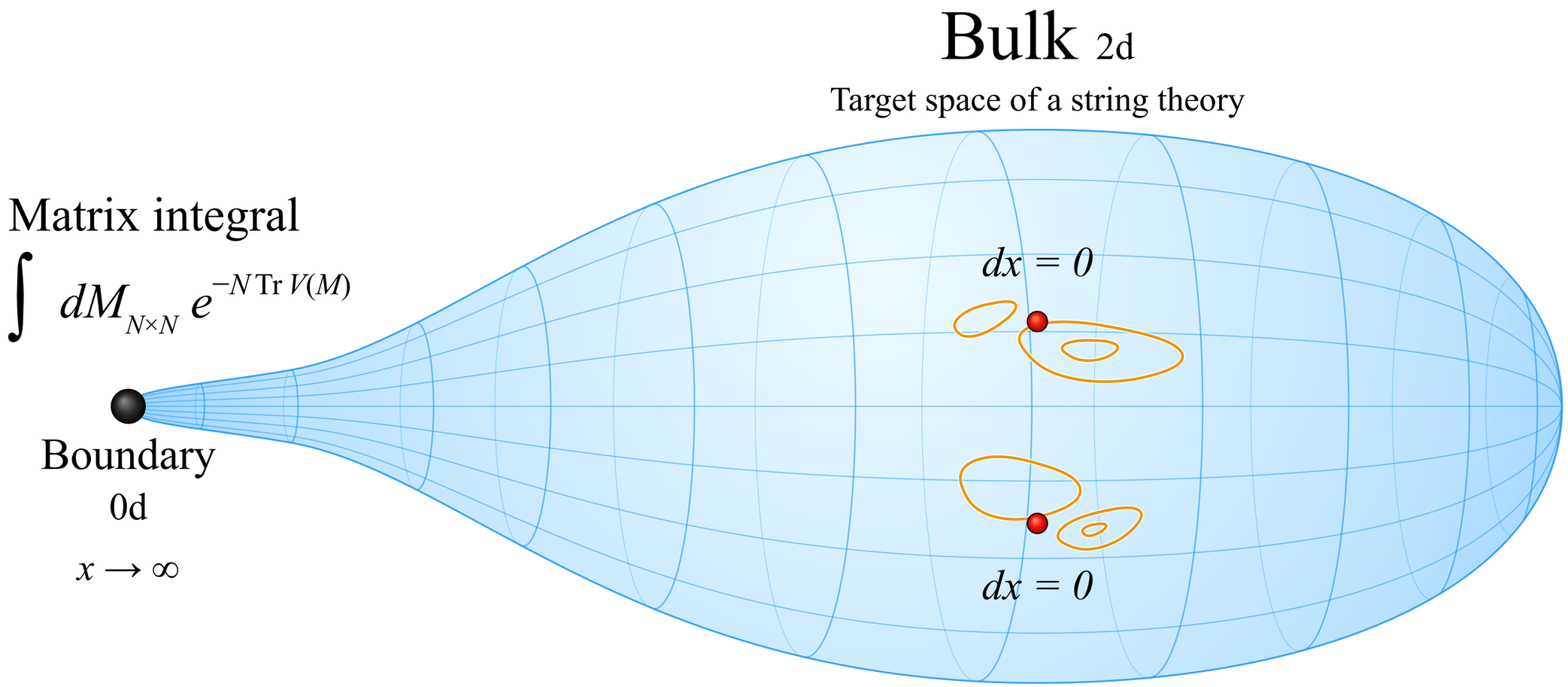
Should be related to Dijkgraaf-Vafa, although not identical…

Matrix Model Holography



How do we describe this string theory?

Matrix Model Holography

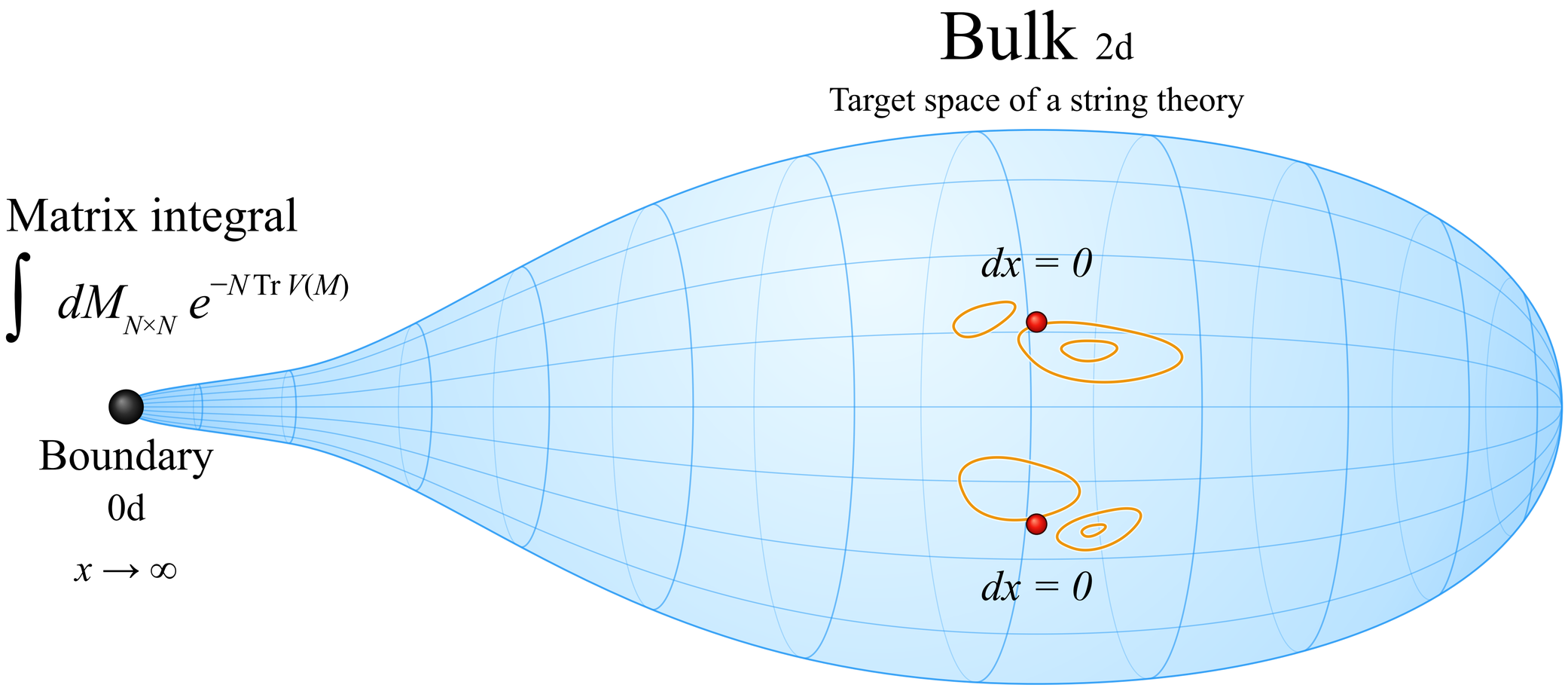


[Bershadsky, Cecotti, Ooguri, Vafa ('93); Costello, Li ('12); Li, Li, Saito ('13)]

Target space formulation:

W_{LG}-deformed 2d version of BCOV (extended by descendants)

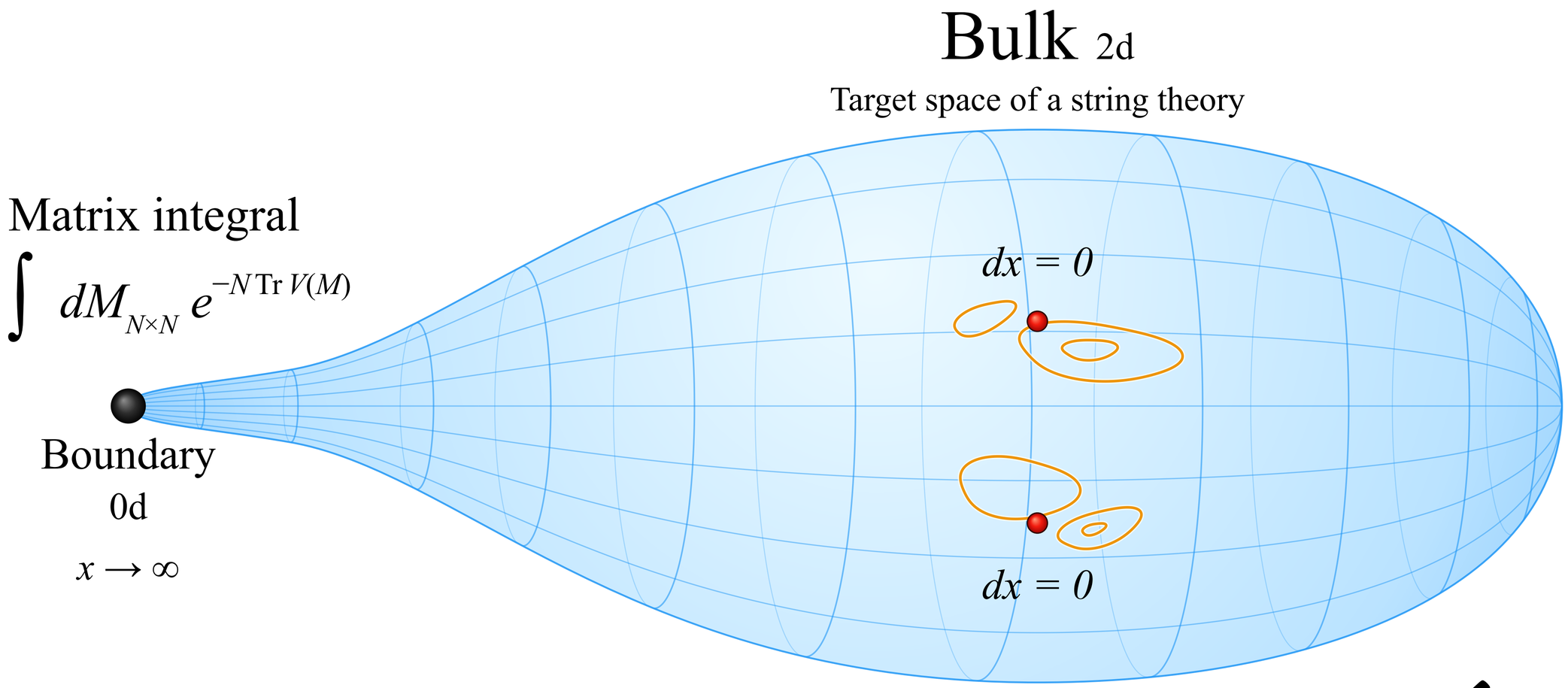
Matrix Model Holography



c.f. bulk polyvector fields in Twisted/Chiral Holography

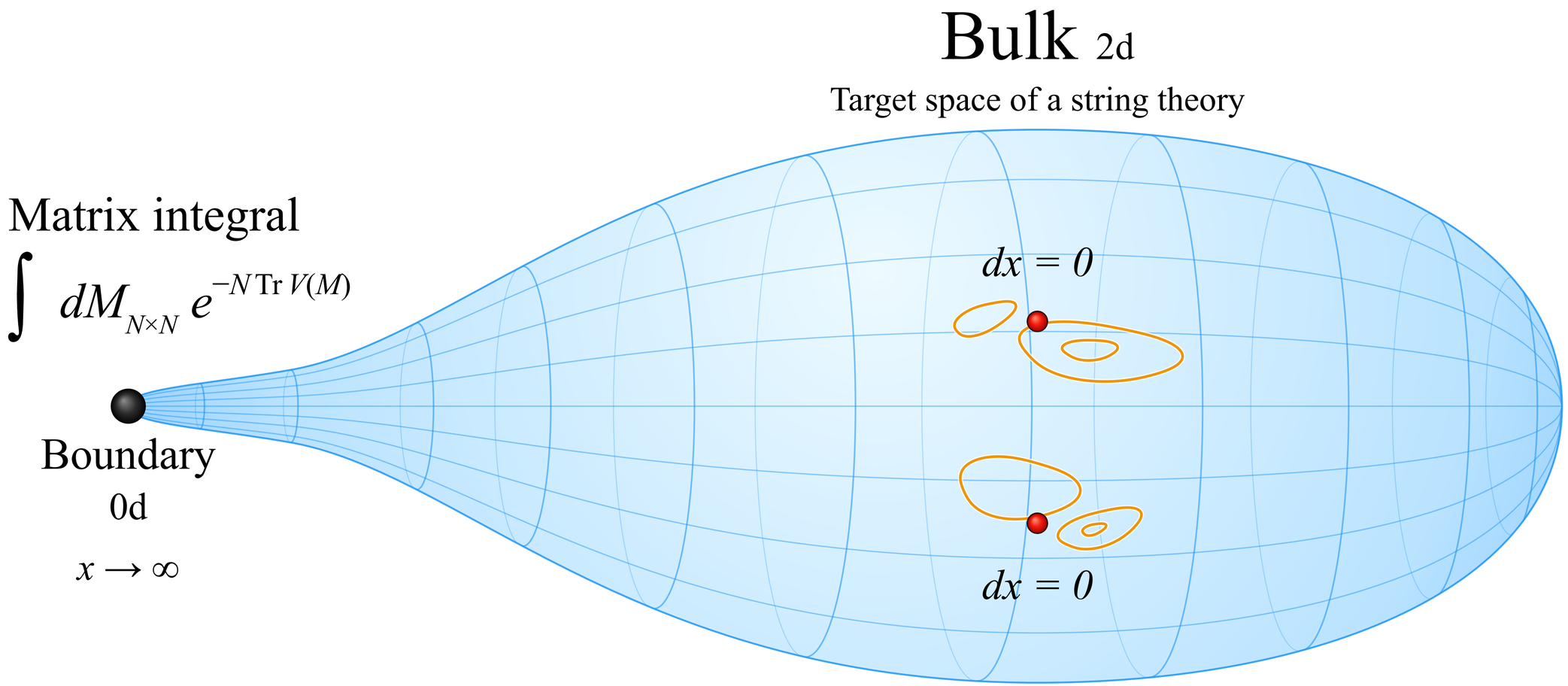
[Costello, Gaiotto (′18); Budzik, Gaiotto, Kulp, Williams, Wu, Yu (′23); Bittleston, Costello, Zeng (′24); Sharma, Skinner (′25)]

Matrix Model Holography



Hard to go to subleading orders in $1/N$

Matrix Model Holography



Our focus: Worldsheet formulation

Matrix Model Holography

Matrix integral

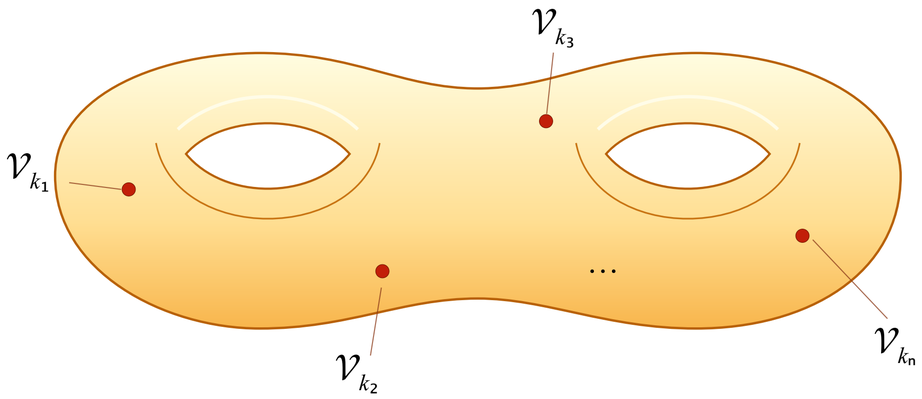
$$\int dM_{N\times N}\, e^{-N \operatorname{Tr} V(M)}$$

Boundary

0d



String worldsheet (also 2d)



Matrix Model Holography

Matrix integral

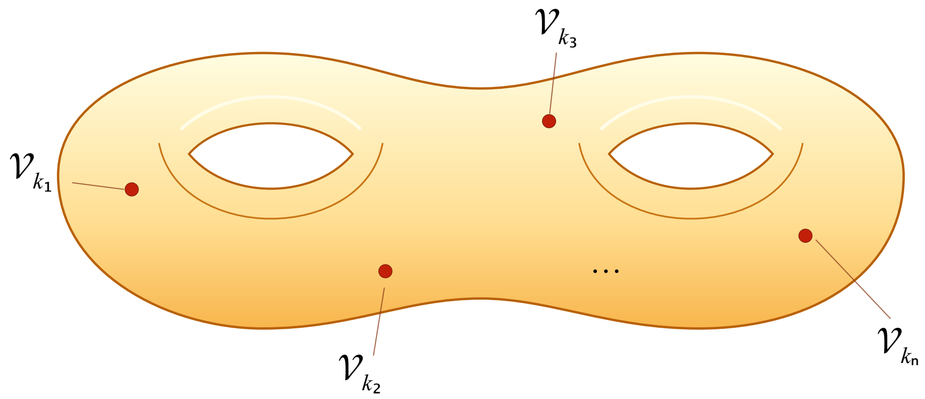
$$\int dM_{N\times N}\, e^{-N\,\mathrm{Tr}\,V(M)}$$

Boundary

0d



String worldsheet (also 2d)



MAIN RESULT

$$\left\langle \prod_{i=1}^{n} Tr(M^{k_i}) \right\rangle_g^c = \int_{\bar{\mathscr{M}}_{g,n}} \left\langle\left\langle \prod_{i=1}^{n} \mathscr{V}_{k_i} \right\rangle\right\rangle_{WS,g}$$

Established to *all orders* in 1/N and *all orders* in the ‘t Hooft coupling(s)!

The Main Result

An exact equivalence of correlators

Our Main Result

A Worldsheet Theory & Integrals over Moduli Space

$$\langle \prod_{i=1}^{n} Tr(M^{k_i}) \rangle_g = \int_{\bar{\mathcal{M}}_{g,n}} \langle\langle \prod_{i=1}^{n} \mathcal{V}_{k_i} \rangle\rangle_{WS,g}$$

$O(N^{2-2g-n})$ ***term in*** $\langle \prod_{i=1}^{n} Tr(M^{k_i}) \rangle$

Our Main Result

A Worldsheet Theory & Integrals over Moduli Space

$$\left\langle \prod_{i=1}^{n} Tr(M^{k_i}) \right\rangle_g = \int_{\bar{\mathcal{M}}_{g,n}} \left\langle \left\langle \prod_{i=1}^{n} \mathcal{V}_{k_i} \right\rangle \right\rangle_{WS,g}$$

1/N expansion = genus expansion

Our Main Result

A Worldsheet Theory & Integrals over Moduli Space

$$\left\langle \prod_{i=1}^{n} Tr(M^{k_i}) \right\rangle_g = \int_{\bar{\mathcal{M}}_{g,n}} \left\langle \left\langle \prod_{i=1}^{n} \mathcal{V}_{k_i} \right\rangle \right\rangle_{WS,g}$$

Finite ‘t Hooft coupling(s)!

Our Main Result

A Worldsheet Theory & Integrals over Moduli Space

$$\left\langle \prod_{i=1}^{n} Tr(M^{k_i}) \right\rangle_g = \int_{\bar{\mathcal{M}}_{g,n}} \left\langle\left\langle \prod_{i=1}^{n} \mathscr{V}_{k_i} \right\rangle\right\rangle_{WS,g}$$



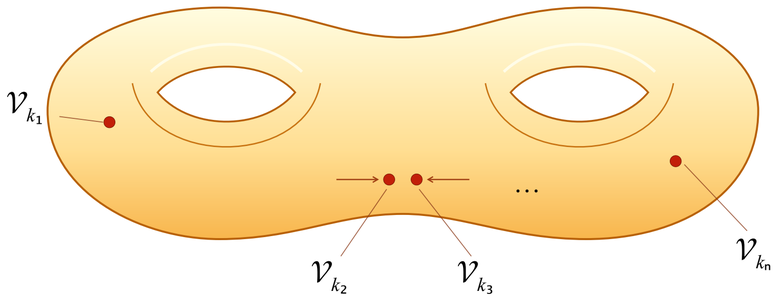
Can formulate for any large N saddle of any 1-matrix integral (single trace potential)

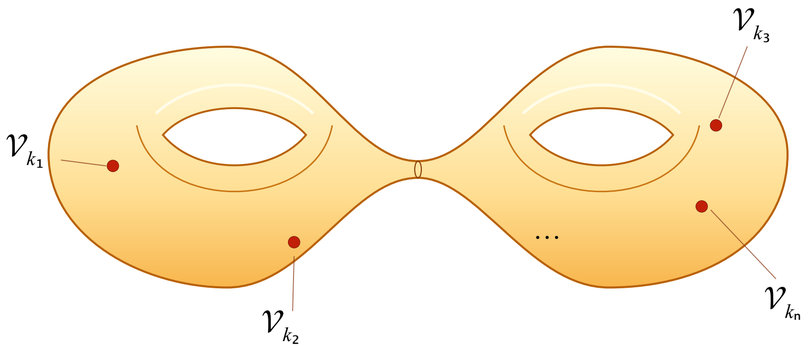
Our Main Result

A Worldsheet Theory & Integrals over Moduli Space

$$\langle \prod_{i=1}^{n} Tr(M^{k_i}) \rangle_g = \int_{\bar{\mathcal{M}}_{g,n}} \langle\langle \prod_{i=1}^{n} \mathcal{V}_{k_i} \rangle\rangle_{WS,g}$$

Includes subtle contact terms on WS





Our Main Result

A Worldsheet Theory & Integrals over Moduli Space

$$\left\langle \prod_{i=1}^{n} Tr(M^{k_i}) \right\rangle_g = \int_{\bar{\mathcal{M}}_{g,n}} \left\langle \left\langle \prod_{i=1}^{n} \mathcal{V}_{k_i} \right\rangle \right\rangle_{WS,g}$$

Explicitly computable!

Our Main Result

A Worldsheet Theory & Integrals over Moduli Space

$$\left\langle \prod_{i=1}^{n} Tr(M^{k_i}) \right\rangle_g = \int_{\bar{\mathcal{M}}_{g,n}} \left\langle\left\langle \prod_{i=1}^{n} \mathscr{V}_{k_i} \right\rangle\right\rangle_{WS,g}$$

Explicitly computable!

***Integrand:* mathematical toolkit of Givental-Teleman reconstruction**

[Givental ('01); Teleman ('12)]

(WIP on physical worldsheet derivation of this algorithm)

Our Main Result

A Worldsheet Theory & Integrals over Moduli Space

$$\left\langle \prod_{i=1}^{n} Tr(M^{k_i}) \right\rangle_g = \int_{\bar{\mathscr{M}}_{g,n}} \left\langle\left\langle \prod_{i=1}^{n} \mathscr{V}_{k_i} \right\rangle\right\rangle_{WS,g}$$

Explicitly computable!

Integrand: ***mathematical toolkit of Givental-Teleman reconstruction***

Integral: ***intersection theory on*** $\mathscr{M}_{g,n}$ ***solved via integrability***

[Witten ('91); Kontsevich ('92)]

Our Main Result

A Worldsheet Theory & Integrals over Moduli Space

$$\left\langle \prod_{i=1}^{n} Tr(M^{k_i}) \right\rangle_g = \int_{\bar{\mathscr{M}}_{g,n}} \left\langle \left\langle \prod_{i=1}^{n} \mathscr{V}_{k_i} \right\rangle \right\rangle_{WS,g}$$

An Example:

$$\langle TrM^{2k} \rangle_{g=1}^{quartic} = \int_{\overline{\mathscr{M}}_{1,1}} \left(\frac{16k-1}{8\gamma} \psi_1 - \frac{3(1-38t_4\gamma^2)}{8\gamma(1-6t_4\gamma^2)} \kappa_1 - \frac{1}{8\gamma} [\partial \mathscr{M}_{1,1}] \right) \frac{\gamma^{2k-1}}{1-6t_4\gamma^2} \frac{(2k)!}{(k-1)!k!}$$

$$= \left(\frac{k-1}{12} - t_4\gamma^2 \frac{k-2}{2} \right) \frac{\gamma^{2k-2}}{1-12t_4} \frac{(2k)!}{(k-1)!k!}$$

$$\gamma^2 \equiv \frac{1-\sqrt{1-12t_4}}{6t_4}$$

Our Main Result

A Worldsheet Theory & Integrals over Moduli Space

$$\left\langle \prod_{i=1}^{n} Tr(M^{k_i}) \right\rangle_g = \int_{\bar{\mathscr{M}}_{g,n}} \left\langle\left\langle \prod_{i=1}^{n} \mathscr{V}_{k_i} \right\rangle\right\rangle_{WS,g}$$

Many more sanity checks -
now automated as a computer code

https://cocalc.com/agiacche/ws-duals-1-mm/demo

Our Main Result

Explicit Worldsheet Theory & Computable Integrals over Moduli Space

$$\langle \prod_{i=1}^{n} Tr(M^{k_i}) \rangle_g = \int_{\bar{\mathscr{M}}_{g,n}} \langle\langle \prod_{i=1}^{n} \mathscr{V}_{k_i} \rangle\rangle_{WS,g}$$

Dual String: ***B-twisted LG model coupled to 2d topological gravity***

Matter Theory

~ akin to ghost system

[Witten ('88) ; Vafa ('90); Verlinde^2 ('92)]

Our Main Result

Explicit Worldsheet Theory & Computable Integrals over Moduli Space

$$\left\langle \prod_{i=1}^{n} Tr(M^{k_i}) \right\rangle_g = \int_{\bar{\mathscr{M}}_{g,n}} \left\langle\left\langle \prod_{i=1}^{n} \mathscr{V}_{k_i} \right\rangle\right\rangle_{WS,g}$$

Dual String: ***B-twisted LG model coupled to 2d topological gravity***

« matter primaries »

$$\mathcal{O}_\alpha \otimes \psi^k$$

« gravitational descendants »
(2k-forms on moduli space)

[Witten ('88); Vafa ('90); Verlinde^2 ('92); Witten ('92) Hanany, Oz, Plesser ('94); Ghoshal, Vafa('95)]

Our Main Result

Explicit Worldsheet Theory & Computable Integrals over Moduli Space

$$\left\langle \prod_{i=1}^{n} Tr(M^{k_i}) \right\rangle_g = \int_{\bar{\mathscr{M}}_{g,n}} \left\langle\left\langle \prod_{i=1}^{n} \mathscr{V}_{k_i} \right\rangle\right\rangle_{WS,g}$$

Dual String: ***B-twisted LG model coupled to 2d topological gravity***

Quasi-Universal Operator Dictionary:

$$TrM^k \leftrightarrow \mathscr{V}_k \equiv \sum_{\alpha\in+,-} \sum_{d=0}^{k-1} c^k_{\alpha,d}\ \mathcal{O}_\alpha \otimes \psi^d$$

Why the correlators always agree

Both obey topological recursion

Spectral Curve (x,y)

Matrix Model

$$\omega_{g,n} = \langle \prod_{i=1}^{n} Tr\left(\frac{dx(z_i)}{x(z_i) - M}\right) \rangle_g$$

=

Origin of Operator Dictionary

String Theory

$$\omega_{g,n} = \int_{\bar{\mathcal{M}}_{g,n}} \langle\langle \prod_{i=1}^{n} \sum_{\alpha_i \in \pm} \mathcal{O}_{\alpha_i} \sum_{d_i} \psi_i^{d_i} d\xi(z_i)_{\alpha_i, d_i} \rangle\rangle_g$$

[Chekov, Eynard, Orantin ('06); Eynard ('11,12); Dunin-Barkowski-Orantin-Shadrin-Spitz ('14), *WIP*]

A few more details

Matrix model spectral curve & the B-model action

From the Matrix Model to the WS

Matrix Model Spectral Curve Data

$$0 = \int dM \frac{\partial}{\partial M_{ij}} \left[\left(\frac{1}{x - M} \right)_{ji} e^{-NTrV(M)} \right] \longrightarrow y^2 - V'(x)y + P(x) = 0$$

$$y = \langle Tr \left(\frac{1}{x - M} \right) \rangle_{g=0}$$

From the Matrix Model to the WS

Matrix Model Spectral Curve Data

$$0 = \int dM \frac{\partial}{\partial M_{ij}} \left[\left(\frac{1}{x - M} \right)_{ji} e^{-NTrV(M)} \right]$$

$$y = \langle Tr \left(\frac{1}{x - M} \right) \rangle_{g=0}$$

$$y^2 - V'(x)y + P(x) = 0$$

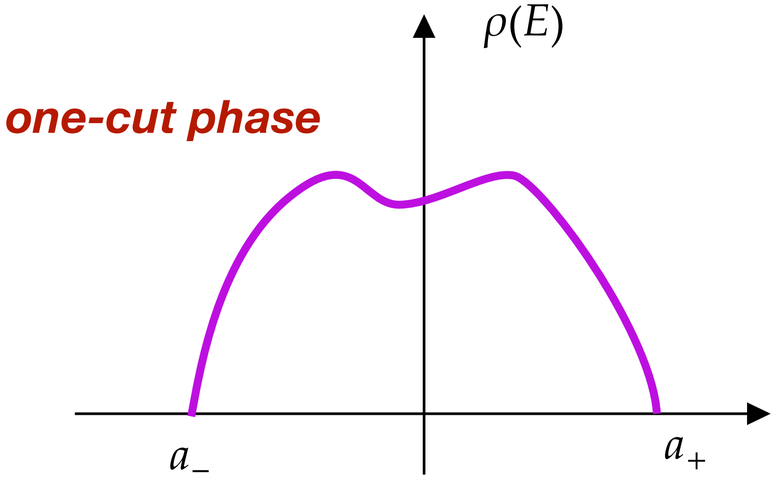
Emergent bulk geometry depends non-perturbatively on couplings in $V(M)$

From the Matrix Model to the WS

Matrix Model Spectral Curve Data

$$0 = \int dM \frac{\partial}{\partial M_{ij}} \left[\left(\frac{1}{x - M} \right)_{ji} e^{-NTrV(M)} \right] \longrightarrow y^2 - V'(x)y + P(x) = 0$$

$$y = \langle Tr \left(\frac{1}{x - M} \right) \rangle_{g=0}$$



one-cut phase

$$x(z) = \gamma \left(z + \frac{1}{z} \right) + \delta$$

(Quasi) Universal

$$y(z) = \sum_k u_k z^k$$

Depends on $V(M)$

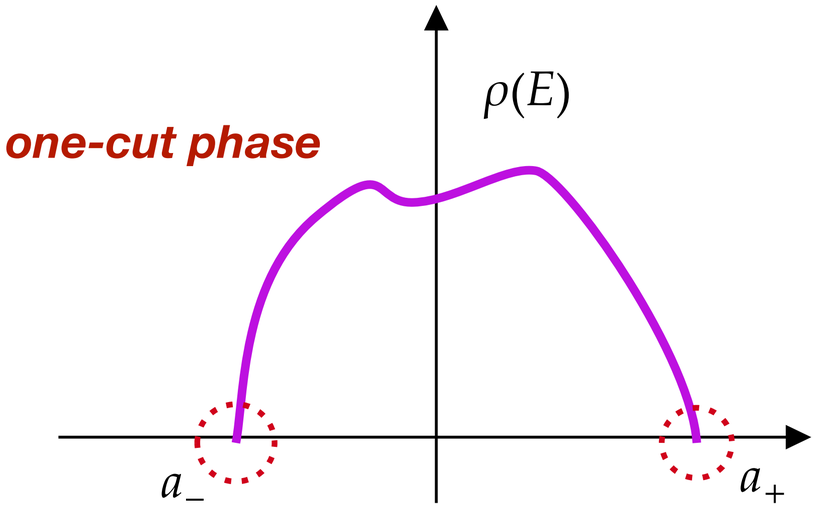
From the Matrix Model to the WS

Matrix Model Spectral Curve Data

$$0 = \int dM \frac{\partial}{\partial M_{ij}} \left[\left(\frac{1}{x - M} \right)_{ji} e^{-NTrV(M)} \right]$$

$$y = \langle Tr \left(\frac{1}{x - M} \right) \rangle_{g=0}$$

$$y^2 - V'(x)y + P(x) = 0$$



one-cut phase

$$x(z) = \gamma \left(z + \frac{1}{z} \right) + \delta$$

(Quasi) Universal

$$dx(z) = 0 \leftrightarrow z = \pm 1$$

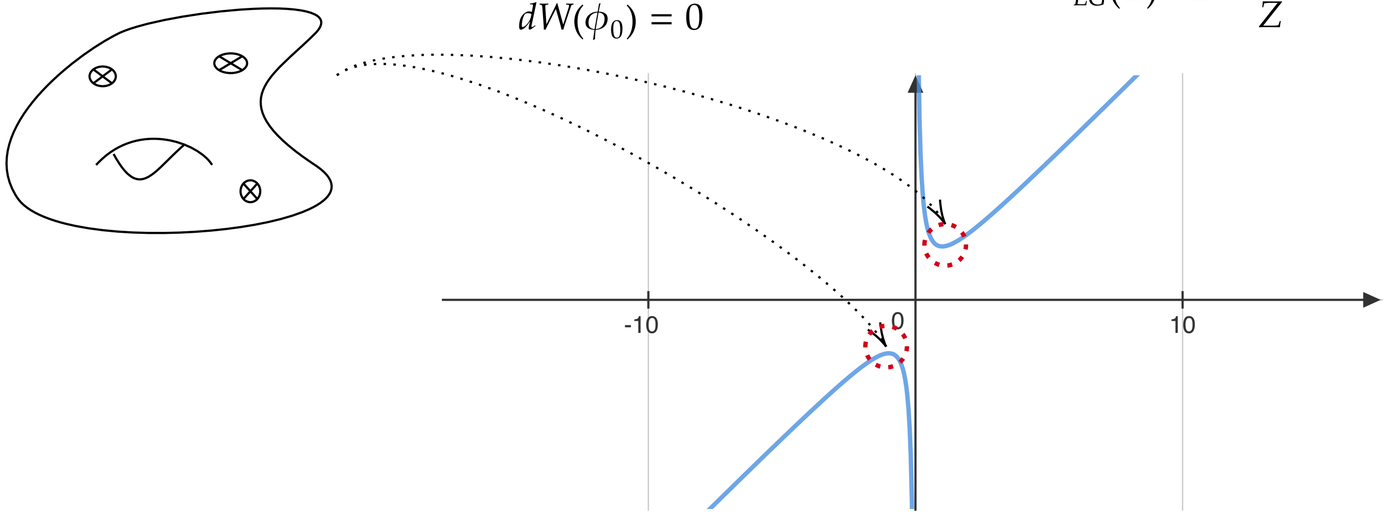
Zeroes of dx

The WS Matter Theory

B-Twisted LG Models (Vafa, ‘90)

$$S_{LG} = \int_{WS} d^2\sigma \; G_{\phi\bar{\phi}} |\, \partial\phi \,|^2 + |\, \partial_\phi W(\phi) \,|^2 + \rho\bar{\partial}\chi + \bar{\rho}\partial\bar{\chi} + \rho\partial_\phi^2 W\bar{\rho} + \bar{\chi}\partial_{\bar{\phi}}^2 \bar{W}\chi$$

Path Integral Localizes!



WS gets mapped into critical points of LG potential

The WS Matter Theory

B-Twisted LG Models (Vafa, ‘90)

$$S_{LG} = \int_{WS} d^2\sigma \; G_{\phi\bar{\phi}} \, | \, \partial\phi \, |^2 + | \, \partial_\phi W(\phi) \, |^2 + \rho\bar{\partial}\chi + \bar{\rho}\partial\bar{\chi} + \rho\partial_\phi^2 W\bar{\rho} + \bar{\chi}\partial_{\bar{\phi}}^2 \bar{W}\chi$$

Dictionary: $W(\phi) \leftrightarrow x(\phi)$ $\qquad \Omega = dy$

Our Relation to Matrix Model Spectral Curve

The WS Matter Theory

B-Twisted LG Models (Vafa, ‘90)

$$S_{LG} = \int_{WS} d^2\sigma \; G_{\phi\bar{\phi}} \, | \, \partial\phi \, |^2 + | \, \partial_\phi W(\phi) \, |^2 + \rho\bar{\partial}\chi + \bar{\rho}\partial\bar{\chi} + \rho\partial_\phi^2 W\bar{\rho} + \bar{\chi}\partial_{\bar{\phi}}^2 \bar{W}\chi$$

Dictionary: $W(\phi) \leftrightarrow x(\phi)$ $\quad \Omega = dy$

Quasi-Universal:

$$W = \gamma(\phi + 1/\phi) + \delta$$

Details of $V(M)$***= geometry of target***

More details on WS theory & correlators tomorrow…

Rajesh's Talk, 10:15 AM

INSTEAD

What is the connection to celestial holography?

Connection to Celestial Holography

[Costello, Paquette (′22); Costello, Paquette, Sharma (′22,23);
Bittleston, Costello, Zeng (′24)]

Our matrix model duality applied to GUE seems to be a subsector of twisted holography!

[Dijkgraaf, Vafa (′02); Budzik, Gaiotto (′21); Jarov(25′);
conversations with K. Budzik, D. Sarkar, A. Sharma]

Connection to Celestial Holography

[Costello, Paquette ('22); Costello, Paquette, Sharma ('22,23); Bittleston, Costello, Zeng ('24)]

2d chiral algebra

($U(N)$ gauged $\beta\gamma$-system)



[Costello, Gaiotto ('18)]

B-Model on

$$SL(2,\mathbb{C}) \sim EAdS_3 \times S^3$$

Connection to Celestial Holography

2d chiral algebra



B-Model on

$$SL(2,\mathbb{C}) \sim EAdS_3 \times S^3$$

[Budzik, Gaiotto (′21)]



twist by

$$Q_{loc}$$

Gaussian matrix model

Connection to Celestial Holography

2d chiral algebra



B-Model on

$$SL(2,\mathbb{C}) \sim EAdS_3 \times S^3$$

[Budzik, Gaiotto (′21)]



twist by

Q_{loc}

add a bulk superpotential W_{loc}

[Costello,Gaiotto (18′); Jarov (′25); Balisa, Casali (′26)]



Gaussian matrix model

Connection to Celestial Holography

2d chiral algebra ⟷ **B-Model on** $SL(2,\mathbb{C}) \sim EAdS_3 \times S^3$

twist by Q_{loc} ↓

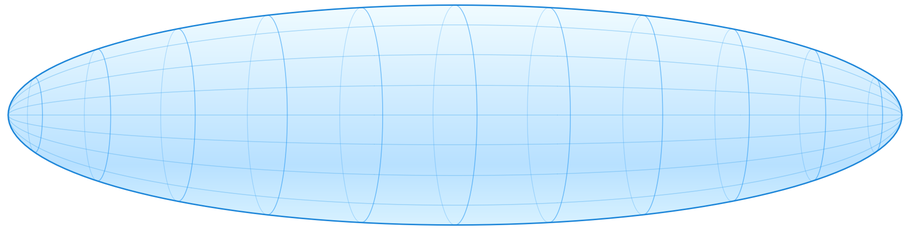
add a bulk superpotential W_{loc} ↓

GUE

$$\int dM e^{-\frac{N}{2} TrM^2}$$

⟷

B-twisted LG on



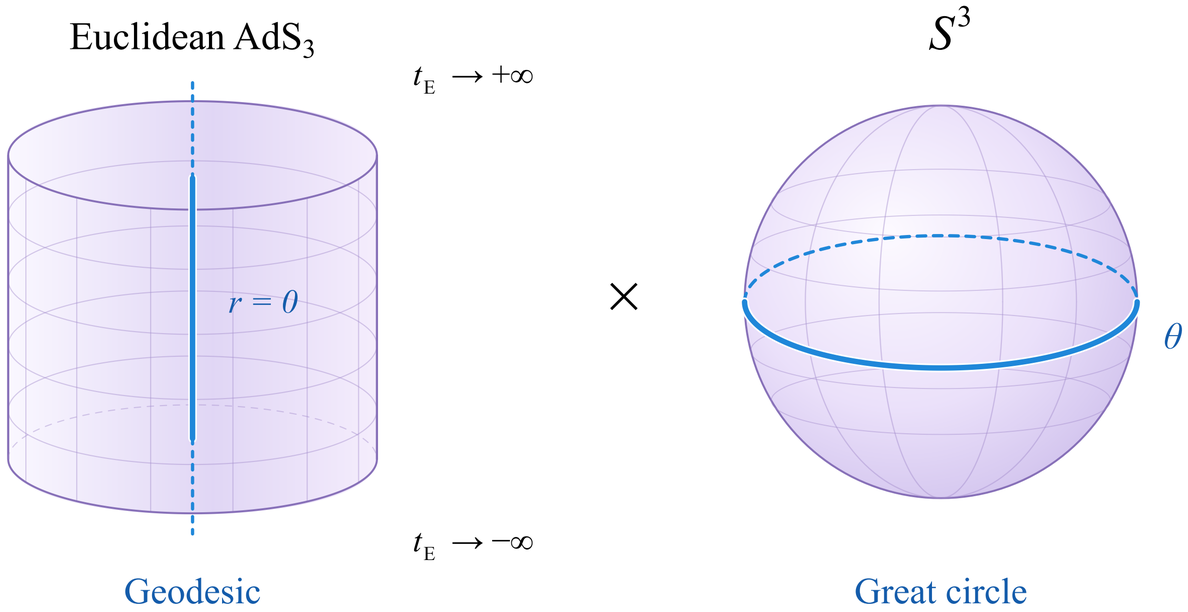
$W_{LG} = x$ GUE spectral curve $y^2 - xy + 1 = 0$ $\Omega = dy$

Connection to Celestial Holography

$$g = \begin{pmatrix} u & y \\ y - x & v \end{pmatrix} \in SL(2,\mathbb{C})$$

$$\det g = 1 \leftrightarrow uv - (y^2 - xy + 1) = 0$$

$$\Omega_{SL(2,\mathbb{C})} = \frac{dy \wedge du \wedge dv}{y}$$



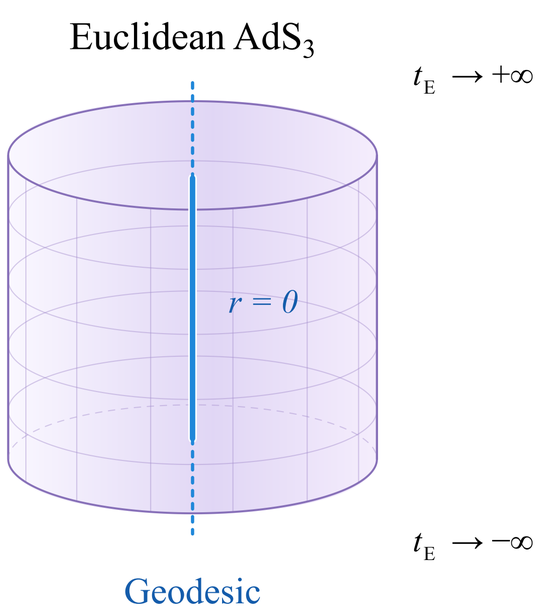
Spectral curve of GUE

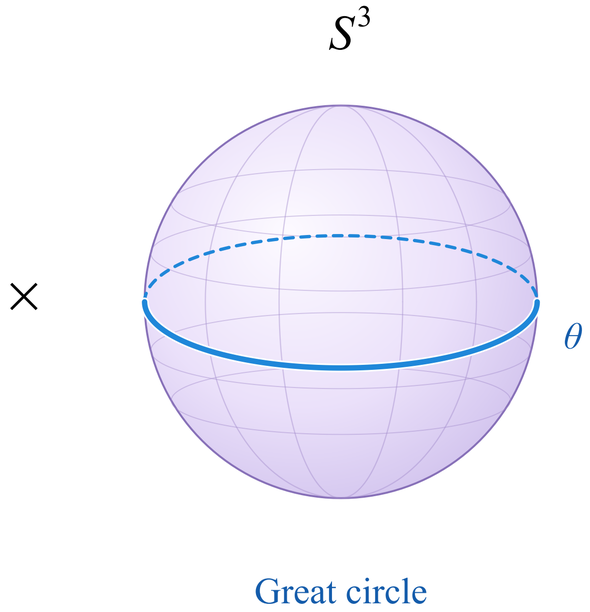
[Dijkgraaf, Vafa (′02); Budzik, Gaiotto (′21)]

Connection to Celestial Holography

$$g = \begin{pmatrix} u & y \\ y - x & v \end{pmatrix} \in SL(2,\mathbb{C})$$

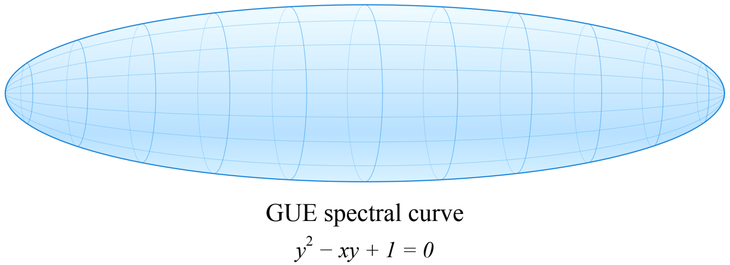
$$\det g = 1 \leftrightarrow uv - (y^2 - xy + 1) = 0$$





Spectral curve of GUE

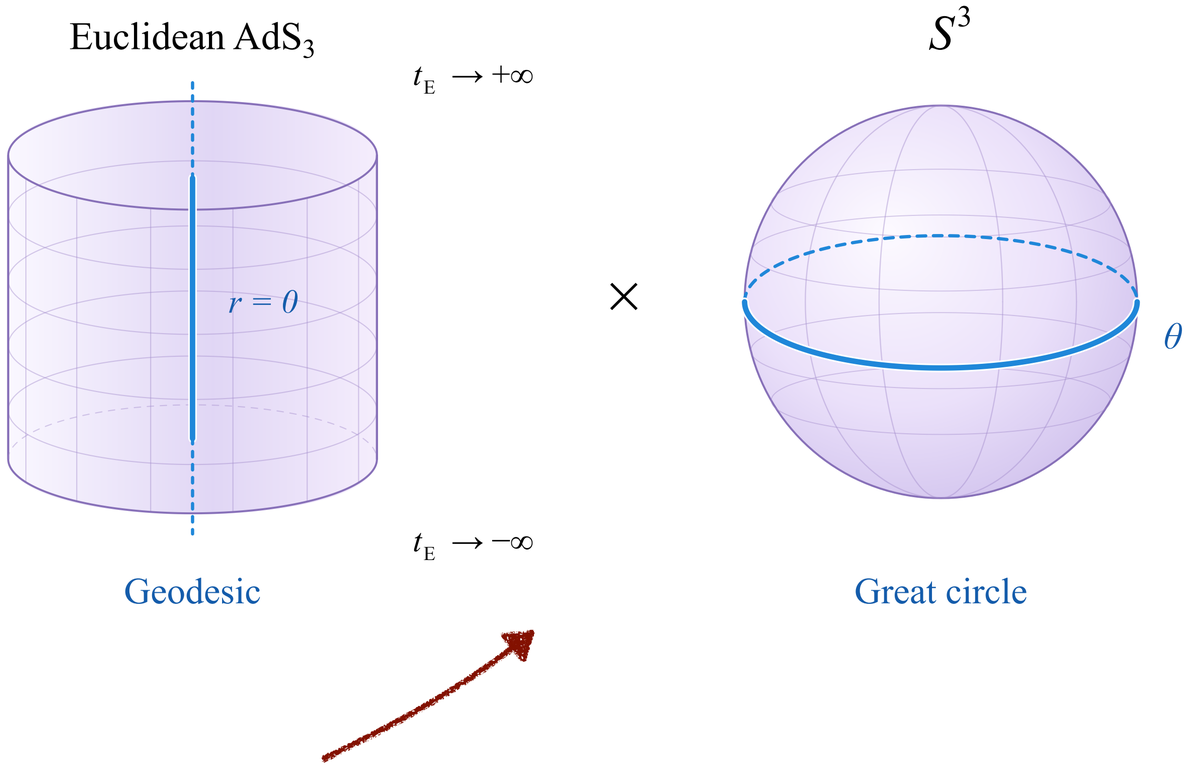
$(u = v = 0)$



Connection to Celestial Holography

$$g = \begin{pmatrix} u & y \\ y - x & v \end{pmatrix} \in SL(2,\mathbb{C})$$

$$\det g = 1 \leftrightarrow uv - (y^2 - xy + 1) = 0$$



Localize to this subspace?

Spectral curve of GUE

$(u = v = 0)$

Connection to Celestial Holography

2d chiral algebra



B-Model on

$$SL(2,\mathbb{C}) \sim EAdS_3 \times S^3$$

$$Q_{loc} = \oint \frac{ds}{2\pi i} \mathrm{Tr}[b(s)\big(X(s) + sY(s)\big)]$$

[Budzik, Gaiotto (′21)]

[*using* Costello, Gaiotto (′18)]



$$W_{loc}(g) = g_{12} - g_{21} = x$$

$$g = \begin{pmatrix} u & y \\ y - x & v \end{pmatrix} \in SL(2,\mathbb{C})$$

Connection to Celestial Holography

2d chiral algebra ⟷ **B-Model on** $SL(2,\mathbb{C}) \sim EAdS_3 \times S^3$

twist by

$$Q_{loc} = \oint \frac{ds}{2\pi i} \mathrm{Tr}[b(s)\big(X(s) + sY(s)\big)]$$

add a bulk superpotential

$$W_{loc} = x = y + \frac{1}{y} - \frac{uv}{y}$$

Gaussian matrix model

localize to $u = v = 0$ **!**

$$W_{LG} = y + \frac{1}{y} \qquad \Omega = dy$$

Connection to Celestial Holography

2d chiral algebra ⟷ **B-Model on** $SL(2,\mathbb{C}) \sim EAdS_3 \times S^3$

twist by Q_{loc} ↓

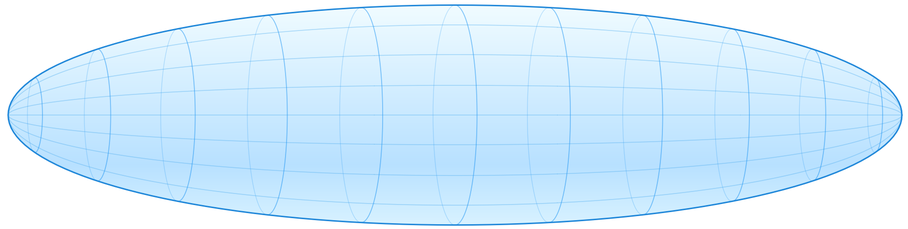
add a bulk superpotential W_{loc} ↓

GUE

$$\int dM e^{-\frac{N}{2} TrM^2}$$

⟷

B-twisted LG on



$W_{LG} = x$ GUE spectral curve $y^2 - xy + 1 = 0$ $\Omega = dy$

Connection to Celestial Holography

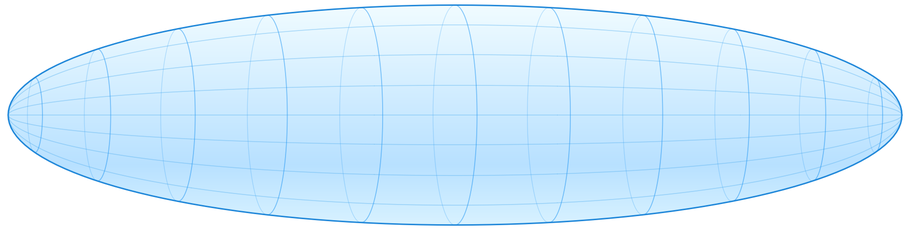
B-Model on

$$SL(2,\mathbb{C}) \sim EAdS_3 \times S^3$$

add a bulk superpotential W_{loc}

Perhaps this is the reduction from DV's matrix model/CY3 to our LG picture?

B-twisted LG on



$W_{LG} = x$ GUE spectral curve $y^2 - xy + 1 = 0$ $\Omega = dy$

Summary: a String Dual for the 1-Matrix Model

- **Standard ‘t Hooft limit, no double scaling**
- **Explicit string dual is B-twisted LG model coupled to 2d topological gravity**
- **Spectral curve dictates worldsheet string action:** $x(z) = W_{LG}(z)$ **&** $\Omega = dy$
- **Precise dictionary between single trace « operators » and vertex operators on the worldsheet:** $TrM^k \leftrightarrow \sum_{\alpha \in +,-} \sum_{d=0}^{k-1} c^k_{\alpha,d}\, \mathcal{O}_\alpha \psi^d$
- **CohFT technology: reduce to explicit, computable integrals over** $\bar{\mathscr{M}}_{g,n}$**!**
- **Correlators agree to all orders in 1/N and all orders in ‘t Hooft coupling(s)**
- **GUE-duality as subsector of twisted holography**

Thank You!

https://cocalc.com/agiacche/ws-duals-1-mm/demo

A New Perspective on the Double-Scaling Limit

What we were zooming into if the string theory already existed?

$$\Omega = dy \to 0$$
$$\text{where } dW = dx = 0$$

Geometry of Criticality

$$t_4 \to t_c = \frac{1}{12}$$

Cusp singularites at critical points!

Strings on Full Geometry

$$W = \gamma(t_4)\left(\phi + \phi^{-1}\right)$$
$$\Omega = \gamma\left(1 - 3t_4\gamma^2(1+\phi^2)\right)d\phi$$
$$g_s = \frac{1}{N}$$

DSL

$$\epsilon \to 0,\ N \to \infty$$

$$t_4 = \frac{1-\epsilon^2}{12}$$
$$\phi = 1 + \sqrt{\frac{\epsilon}{2}}\tilde{\phi} + O(\epsilon)$$

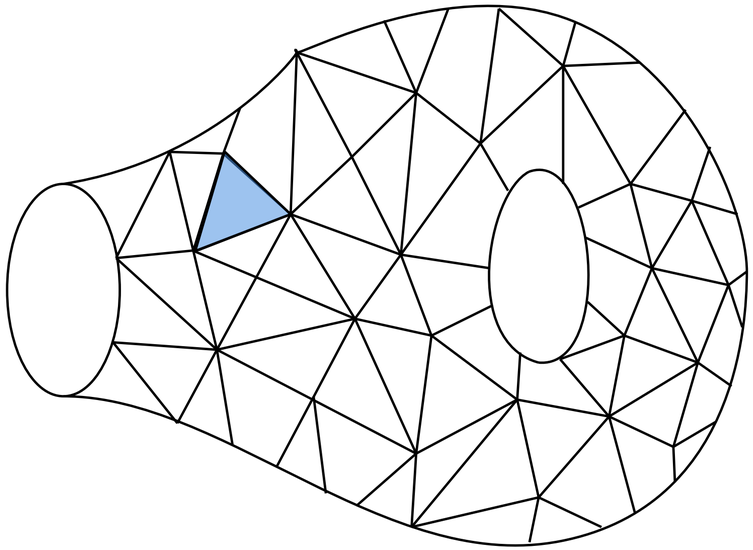
Strings on Blow-Up of Singularity

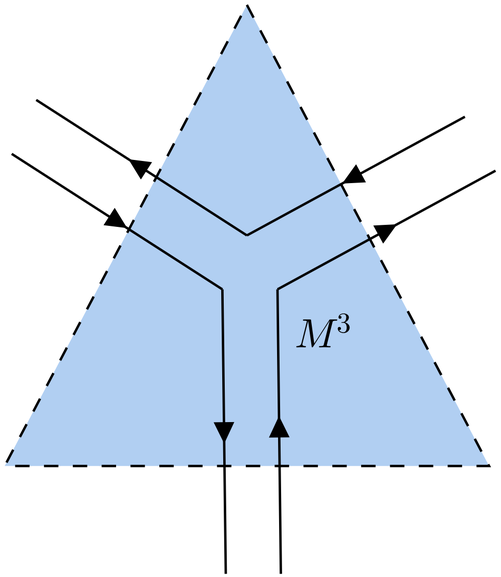
$$\tilde{W} = \frac{1}{\sqrt{2}}(\tilde{\phi}^2 - 2)$$
$$\tilde{\Omega} = (\tilde{\phi}^2 - 1)d\tilde{\phi}$$
$$\tilde{g}_s = \frac{1}{N(t_4 - t_c)^{5/4}}$$

A Past Ideological Detour

(Polyakov, Gross, Migdal, Brézin, Kazakov, Kostov, Shenker, Douglas…)

Feynman Diagrams look like *discretised* worldsheets

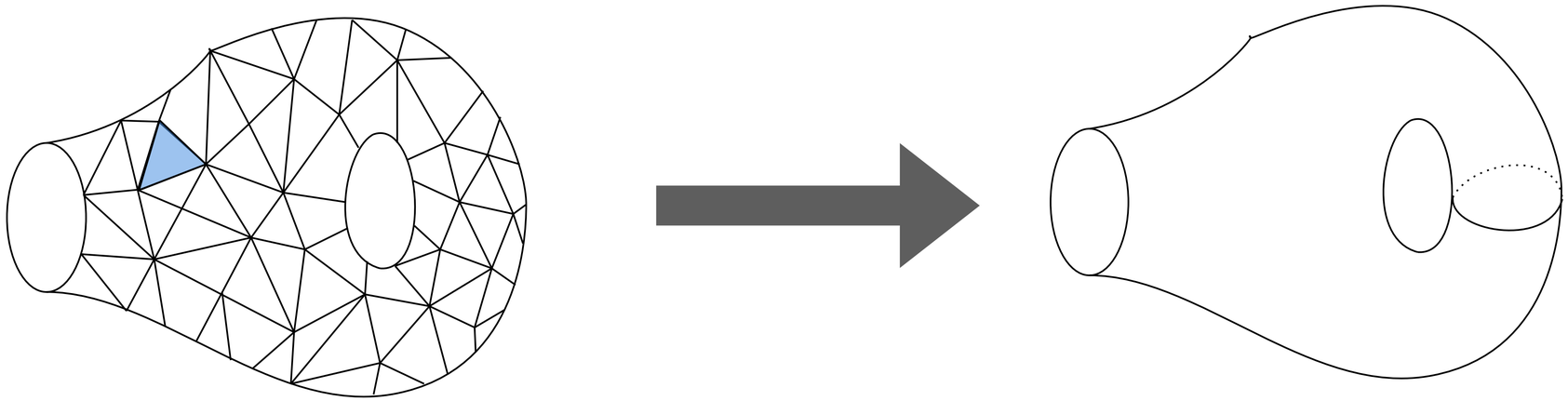




Graph dual of Feynman diagram = triangulation of worldsheet

Unlike AdS/CFT

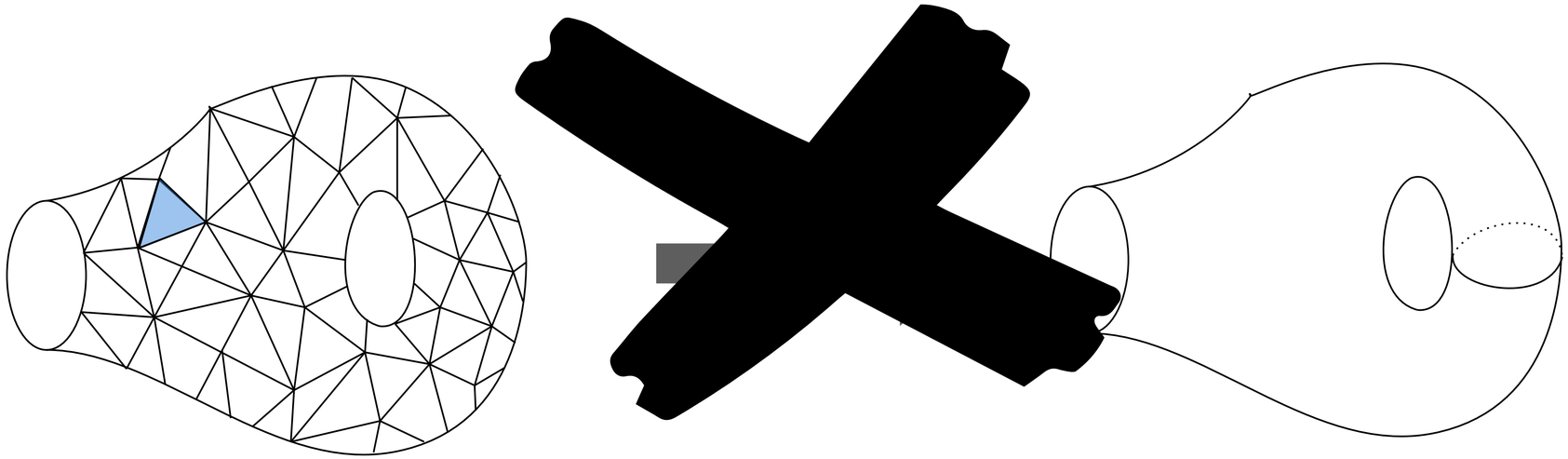
Requires continuum limit on WS/ “ double-scaling” limit on Matrix Model
(*NOT* standard ’t Hooft limit)



“Strings emerge only in limit
of (infinitely) dense Feynman Diagrams”?

Unlike AdS/CFT

Requires continuum limit on WS/ “ double-scaling” limit on Matrix Model
(*NOT* standard ’t Hooft limit)



A Different Picture: map individual Feynman diagrams to specific worldsheets via Strebel Differentials

[Strebel (′84); Gopakumar (′03,04,05); Aharony, Komargodski, Razamat (′06); Razamat (′09); Gopakumar, Kaushik, Komatsu, EAM, Sarkar (′24)]

Pure Matter Theory Correlators

Chiral Ring (BRST)

$$\frac{\mathbb{C}[\phi]}{dW}$$

Ring Relation

$$d(\phi + \frac{1}{\phi}) = 0 \leftrightarrow \phi^2 = 1$$

Phys. Ops

$$\{1, \phi\}$$

Can compute from path integral localization

$$\langle \phi^{l_1}(\sigma_1) \dots \phi^{l_n}(\sigma_n) \rangle_g = \phi^{l_1} \dots \phi^{l_n} \left(\frac{\partial^2 W}{\partial y(\phi)^2} \right)^{g-1} |_{\phi=\pm 1}$$

[Witten ('88); Lerche, Warner, Vafa ('89); Hori, Vafa ('00)]

Coupling to Gravity

« CohFTs » as computational tools

General Correlator

$$\int_{\bar{\mathcal{M}}_{g,n}} \langle\langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle\rangle_g \; \psi_1^{d_1} \dots \psi_n^{d_n}$$

CohFT

$$\langle\langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle\rangle_g^{+top.grav} = R \,.\, T \,.\, \langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle_g^{pure\ matter}$$

= Axiomatization of topological strings

[Kontsevich,Manin (′94); Givental (′01,04,07); Teleman (′12); Pandharipande-Pixton-Zvonkine (′13)]

Coupling to Gravity

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General Correlator

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CohFT

$$\langle\langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle\rangle_g^{+top.grav} = R \,.\, T \,.\, \langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle_g^{pure\ matter}$$

Used in previous contexts,

e.g. GW on $\mathbb{P}^1$ and its B-model LG mirror, r-spin class…

Coupling to Gravity

« CohFTs » as computational tools

General Correlator

$$\int_{\bar{\mathcal{M}}_{g,n}} \langle\langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle\rangle_g \, \psi_1^{d_1} \dots \psi_n^{d_n}$$

CohFT

$$\langle\langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle\rangle_g^{+top.grav} = R \,.\, T \,.\, \langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle_g^{pure\ matter}$$

« R-Matrix Action »

depends on x ~ universal

« Translation »

depends on y ~on V(M)

Comparing w/ Matrix Model

$$\langle \prod_{i=1}^{3} Tr(M^{2k_i}) \rangle_{g=0} = \int_{\bar{\mathscr{M}}_{g=0,n=3}} \langle \prod_{i=1}^{n} \mathscr{V}_{2k_i} \rangle_{WS,g}$$

$$\mathscr{V}_{2k} = \sum_{\alpha \in +,-} \sum_{d=0}^{2k-1} c_{\alpha,d}^{2k} \, \mathcal{O}_\alpha \otimes \psi^d$$

Can ignore gravity

$\mathscr{M}_{0,3} \sim pt$

$$\langle \prod_{i=1}^{3} Tr(M^{2k_i}) \rangle_{g=0} = \langle \mathcal{O}_+^3 \rangle_{g=0} \prod_{i=1}^{3} c_{+,0}^{(2k_i)} + \langle \mathcal{O}_-^3 \rangle_{g=0} \prod_{i=1}^{3} c_{-,0}^{(2k_i)}$$

Universal Answer for Planar 3pt Function from LG model

Example: Quartic MM

[Gopakumar,Pius ('14)]

MM perturbation theory

$$\langle \prod_{i=1}^{3} Tr(M^{2k_i}) \rangle_{g=0} = \sum_{s=0}^{\infty} \frac{(3t_4)^s}{s!} \frac{(k_1+k_2+k_3+2s-1)!}{(k_1+k_2+k_3+s-1)!} \times \prod_{i=1}^{3} \frac{(2k_i)!}{k_i!(k_i-1)!}$$

String answer

$$\langle \mathcal{O}_+^3 \rangle_{g=0} \prod_{i=1}^{3} c_{+,0}^{2k_i} + \langle \mathcal{O}_-^3 \rangle_{g=0} \prod_{i=1}^{3} c_{-,0}^{2k_i} = \frac{\gamma^{2(k_1+k_2+k_3)-2}}{\sqrt{1-12t_4}} \prod_{i=1}^{3} \frac{(2k_i)!}{(k_i-1)!k_i!}$$

$$\gamma(t_4) = \frac{1-\sqrt{1-12t_4}}{6t_4}$$

Agreement to all orders in 't Hooft coupling!

Coupling to Gravity

« CohFTs » as computational tools

General Correlator

$$\int_{\bar{\mathcal{M}}_{g,n}} \langle\langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle\rangle_g \, \psi_1^{d_1} \dots \psi_n^{d_n}$$

CohFT

$$\langle\langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle\rangle_g^{+top.grav} = R \,.\, T \,.\, \langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle_g^{pure\ matter}$$

Encodes degenerations of WS

depends on x ~ universal

Encodes choice of Ω

depends on y ~on V(M)

Coupling to Gravity

« CohFTs » as computational tools

General Correlator

$$\int_{\bar{\mathscr{M}}_{g,n}} \langle\langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle\rangle_g \, \psi_1^{d_1} \dots \psi_n^{d_n}$$

CohFT

$$\langle\langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle\rangle_g^{+top.grav} = R \,.\, T \,.\, \langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle_g^{pure\ matter}$$

Fully Explicit

$$T^{\alpha}(u) = u - \sqrt{\frac{uH_{\alpha}}{2\pi}} \int_{\Gamma_{\alpha}} dy e^{-\frac{x - x(\alpha)}{u}}$$

+ similar for R-Matrix

A Worldsheet Take on CohFT

Demystifying CohFTs

$$T^{\alpha}(u) = u - \sqrt{\frac{uH_{\alpha}}{2\pi}} \int_{\Gamma_{\alpha}} dy e^{-\frac{x - x(\alpha)}{u}}$$

Our Dictionary

$$\Omega \leftrightarrow dy$$
$$W \leftrightarrow x$$

[Nekrasov ('18)]

$$\int_{\Gamma\alpha} \Omega e^{-\frac{W - W_{\alpha}}{2u}}$$

Connection between Top. Gravity & S^1-equivariant cohomology

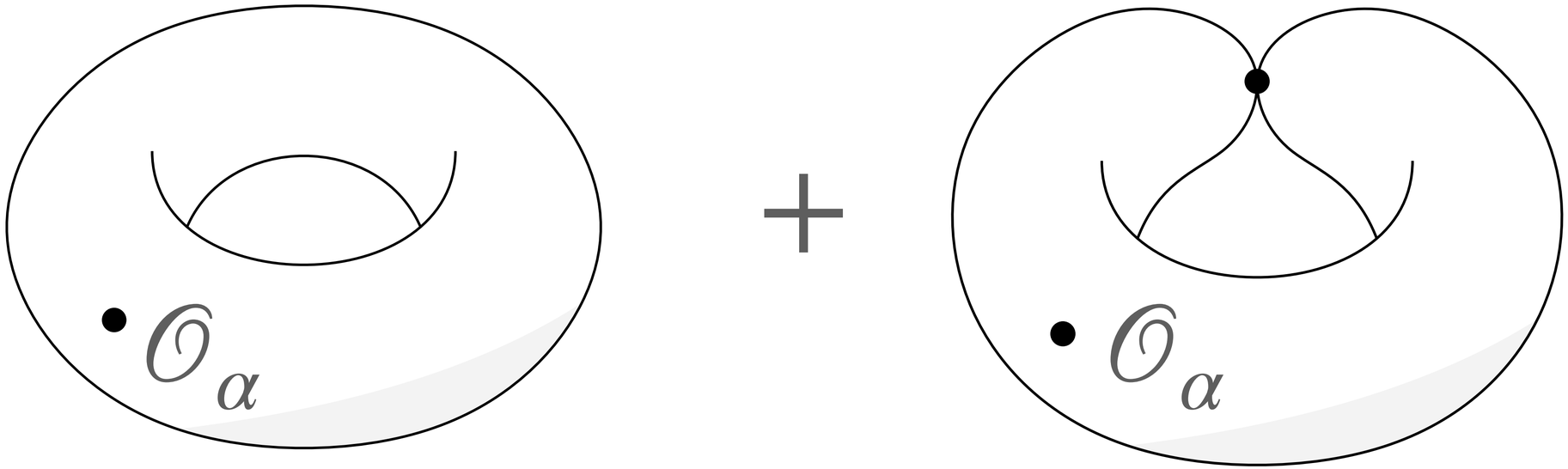
[Getzler ('93); Givental ('06); To appear, w/ Giacchetto]

(S^1—Equivariant) Worldsheet Disc Partition Function

A Worldsheet Feynman Calculus

Example at genus one

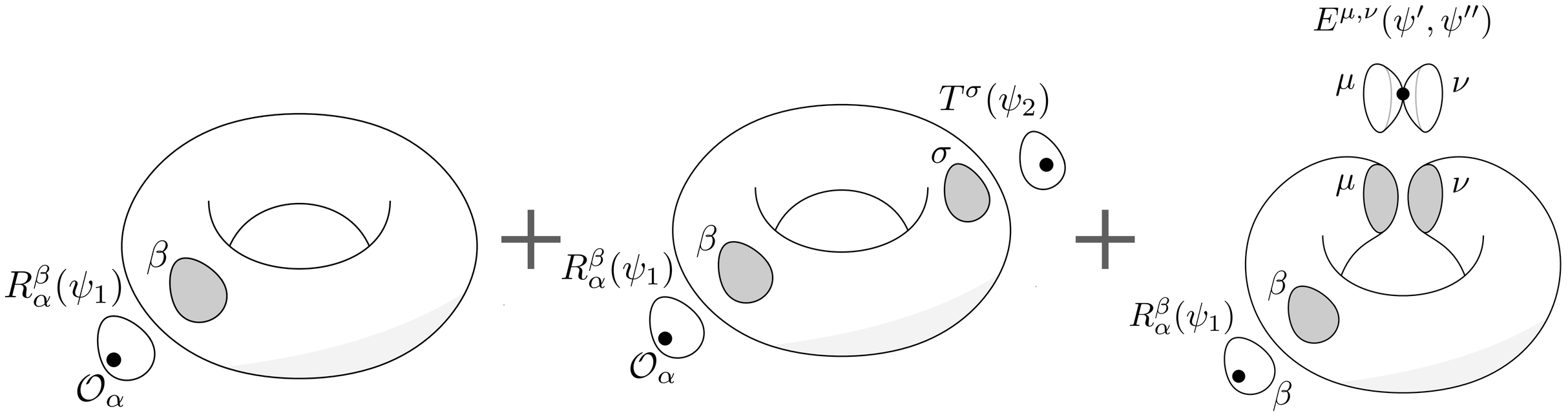
$$\langle\langle \mathcal{O}_\alpha \rangle\rangle_{g=1}^{CohFT}$$



Extra contribution from boundary of moduli space

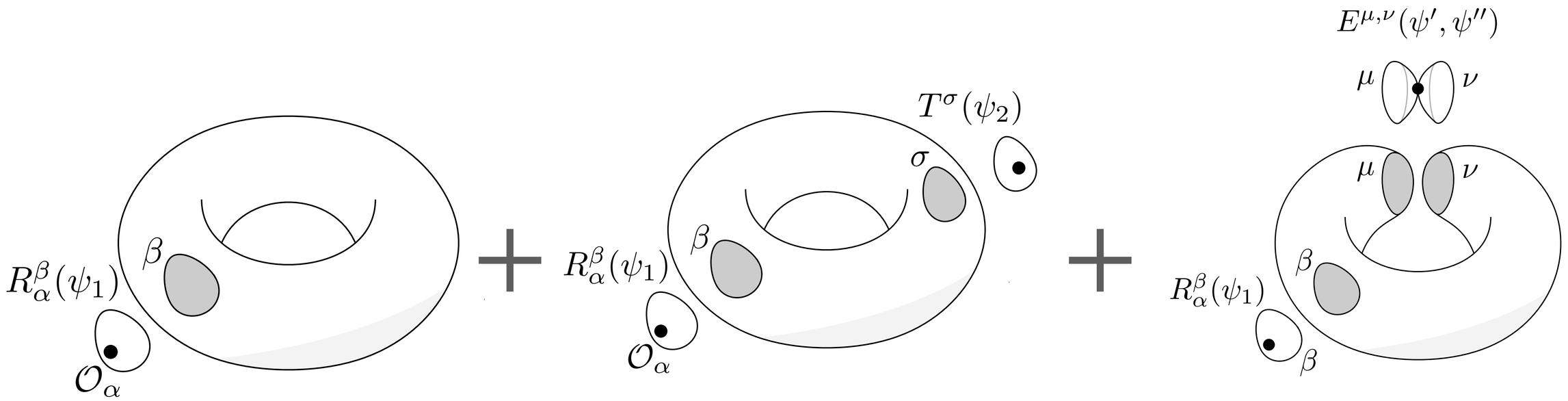
A Worldsheet Feynman Calculus

Example at genus one $\langle\langle \mathcal{O}_\alpha \rangle\rangle_{g=1}^{CohFT}$



A Worldsheet Feynman Calculus

Example at genus one $\langle\langle \mathcal{O}_\alpha \rangle\rangle_{g=1}^{CohFT}$



$$= \sum_{\beta \in \pm} R_\alpha^\beta(\psi) \langle \mathcal{O}_\beta \rangle_{g=1}^{TFT} + \sum_{\beta,\sigma \in \pm} R_\alpha^\beta(\psi) \langle \mathcal{O}_\beta \mathcal{O}_\sigma \rangle_{g=1}^{TFT} T^\sigma(\psi') + \sum_{\beta,\mu,\nu \in \pm} R_\alpha^\beta(\psi) \langle \mathcal{O}_\beta \mathcal{O}_\mu \mathcal{O}_\nu \rangle_{g=0}^{TFT} E^{\mu\nu}(\psi', \psi'')$$

Another MM Comparison

Example at genus one

String answer

$$\langle \mathscr{V}_{2k} \rangle_{g=1} = \sum_{\alpha \in \pm} c_{\alpha,1}^{2k} \langle \mathcal{O}_\alpha \rangle_{g=1}^{TFT} \int_{\bar{\mathscr{M}}_{1,1}} \psi_1 + c_{0,\alpha}^{2k} \int_{\bar{\mathscr{M}}_{1,1}} \langle\langle \mathcal{O}_\alpha \rangle\rangle_{g=1}^{CohFT}$$

= Matrix answer

$$\langle TrM^{2k} \rangle_{g=1} = \left(\frac{k-1}{12} - t_4 \gamma^2 \frac{k-2}{2} \right) \frac{\gamma^{2k-2}}{1-12t_4} \frac{(2k)!}{(k-1)!k!} .$$

(Agreement checked using Non-Pert. Matrix Schwinger-Dyson Eqs)

Sanity Check

& a striking implication

String answer

$$\langle TrM^{2k}\rangle_{g=1} = \left(\frac{k-1}{12} - t_4\gamma^2\frac{k-2}{2}\right)\frac{\gamma^{2k-2}}{1-12t_4}\frac{(2k)!}{(k-1)!k!}\,.$$

Zero Coupling Limit

$$t_4 \to 0 \quad (\gamma \to 1)$$

[Harer,Zagier ('86)]

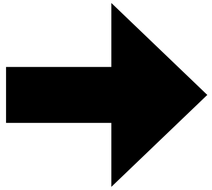
$$\langle TrM^{2k}\rangle^{GUE}_{g=1} = \frac{k-1}{12}\frac{(2k)!}{(k-1)!k!}$$

Gives INTEGERS, reproduces free-field Wick contractions!

Important Take-Away

Back to Feynman diagrams

Reproduces free-field Wick contractions , gives INTEGERS!



Reformulated the combinatorics of gauge-theory Feynman diagrams as integrals over moduli space of Riemann surfaces

``universal seed of gauge/string duality“

A BMN-like Limit

« Universal Subsector »

$$\lim_{\kappa\to\infty}\frac{\left\langle\prod_{i=1}^{n}\operatorname{Tr}M^{\kappa a_i}\right\rangle_g}{(2\gamma)^{\kappa|a|}\kappa^{3g-3+3n/2}}=\left(\frac{\langle\mathcal{O}_+^n\rangle_g^{LG}}{\Omega_+^n}+(-1)^{g-1}\frac{\langle\mathcal{O}_-^n\rangle_g^{LG}}{\Omega_-^n}\right)\times\frac{1}{(2\gamma)^{3g-3+n}}\sum_{m_1,\ldots,m_n\geq 0}\int_{\overline{\mathscr{M}}_{g,n}}\prod_{i=1}^{n}\frac{\psi_i^{m_i}a_i^{m_i+\frac{1}{2}}}{\sqrt{2\pi}}$$

Reflects Airy Universality
(dominated by eigenvalues close to edge)

Reproduce + Generalize Okounkov-Pandharipande (GUE)!

A BMN-like Limit

« Decoupling the Matter Theory »

$$\lim_{\kappa\to\infty}\frac{\langle\prod_{i=1}^{n}\mathrm{Tr}\,M^{\kappa a_i}\rangle_g}{(2\gamma)^{\kappa|a|}\kappa^{3g-3+3n/2}}=\left(\frac{\langle\mathcal{O}_+^n\rangle_g^{TFT}}{\Omega_+^n}+(-1)^{g-1}\frac{\langle\mathcal{O}_-^n\rangle_g^{TFT}}{\Omega_-^n}\right)\times\frac{1}{(2\gamma)^{3g-3+n}}\sum_{m_1,\ldots,m_n\geq 0}\int_{\overline{\mathcal{M}}_{g,n}}\prod_{i=1}^{n}\frac{\psi_i^{m_i}a_i^{m_i+\frac{1}{2}}}{\sqrt{2\pi}}$$

From our operator dictionary

$$\lim_{k\to\infty} Tr\ M^{2k}\sim\mathcal{O}_\alpha\times 2^{2k}\sum_d k^{d+1/2}\psi^d$$

Maximize ψ-class insertions

$\leftrightarrow$

Decouples matter theory

$$\langle O_{\alpha_1}\ldots O_{\alpha_n}\rangle^{TFT}\times\int_{\mathcal{M}_{g,n}}\prod_{i=1}^{n}\sum_d c_{\mathbf{k_i}>>\mathbf{1},d}\ \psi_i^d$$

Pure 2d top gravity in BMN limit!