Horizons and Null Infinity

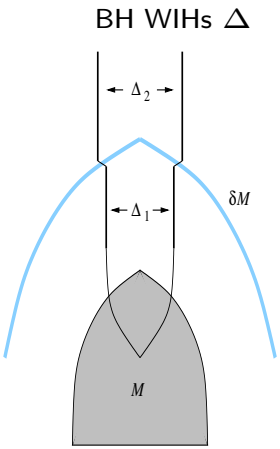
An Unforeseen Interplay between Geometry and Physics

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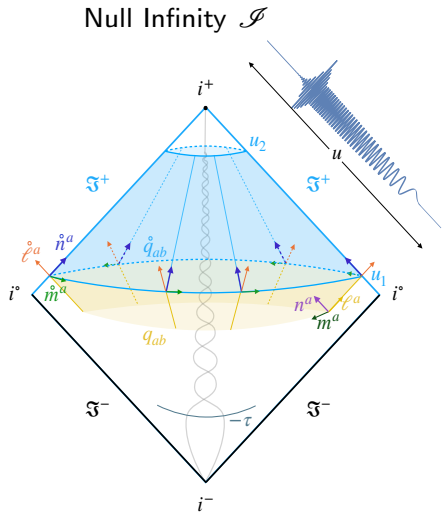
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Preamble



• Horizons, Δ, of black holes in equilibrium and future Null Infinity, $\mathscr{I}^+$, of asymptotically flat space-times are both null 3-manifolds, but with very different physical connotations.



Δ lies in the strong field region and it is quiescent, devoid of any radiation or fluxes. $\mathscr{I}^+$ is the arena for gravitational radiation admits large fluxes & lies in the asymptotic region. Yet, surprisingly, they share a large number of geometric properties, making them both Weakly Isolated Horizons (WIHs) $\mathfrak{h}$. In particular, the BMS group at $\mathscr{I}^+$ descends directly from the symmetry group of WIHs!

• How can they carry drastically different physics, then? Interestingly, one can pinpoint the reason: Einstein's equations hold on Δ, while conformal Einstein's equations hold on $\mathscr{I}$! Consequently, in the fundamental geometrical equation that holds on all WIHs, different terms vanish on Δ and $\mathscr{I}$, leading to almost opposite physical consequences. I will discuss this unforeseen interplay between geometry and physics, emphasizing the underlying conceptual structure.

Organization

1. Geometrical Weakly Isolated Horizons (WIHs)
   (No field equations assumed)
2. Black Hole (and Cosmological) WIHs
   (Einstein's equations $\Rightarrow$ No true DOF)
3. Null Infinity as a WIH
   (Conformal Einstein's equations $\Rightarrow$ True DOF)
4. Interplay between Symmetries
   ($\mathfrak{B}$ on $\mathscr{I}^+$ descends from $\mathfrak{G}$ on WIHs)
5. Summary and Discussion

References for the source material are listed at the end.

1. Geometrical WIHs

• A *Non Expanding Horizon (NEH)* in 4-d Space-time $(\bar{M}, \bar{g}_{ab})$ is a null submanifold $\mathfrak{h}$, topologically $\mathbb{S}^2 \times \mathbb{R}$, such that:

(i) Every null normal $\bar{k}^a$ to $\mathfrak{h}$ is expansion-free, $\theta_{\bar{k}} = 0$; and,

(ii) On $\mathfrak{h}$ the Ricci tensor satisfies $\bar{R}_a{}^b \bar{k}^a = \alpha \bar{k}^b$ for some function α.

• Raychaudhuri Eq. $\Rightarrow$ the intrinsic (degenerate) metric $\bar{q}_{ab}$ satisfies $\mathcal{L}_{\bar{k}} \bar{q}_{ab} = 0$; it is 'time independent'. As a result, by pull-back, the space-time derivative operator $\bar{\nabla}$ induces a canonical intrinsic derivative $\bar{D}$ on $\mathfrak{h}$ $(\bar{\nabla} = \bar{D})$. Furthermore, it satisfies: $\bar{D}_a \bar{q}_{ab} = 0$, and, $\bar{D}_a \bar{k}^b = \bar{\omega}_a \bar{k}^b$ for some $\bar{\omega}_a$ (the 'rotation' or 'Hajicek' 1-form).

• We can always restrict ourselves to geodesic null normals $\bar{k}^a \bar{D}_a \bar{k}^b = 0$. Then NEH conditions $\Rightarrow$ $\mathcal{L}_{\bar{k}} \bar{\omega}_a = 0$; $\bar{\omega}_a$ is also 'time-independent'. Furthermore, we can now severely restrict the rescaling freedom in $\bar{k}^a$ by demanding that $\bar{\omega}_a$ be divergence-free, i.e, $\bar{q}^{ab} \bar{D}_a \bar{\omega}_b = 0$. Only remaining freedom: $\bar{k}^a \to c\bar{k}^a$ where c is a positive constant.

• Thus, every NEH can be naturally equipped with a (small) equivalence class of null normals $[\bar{k}^a]$ (where $\bar{k}'^a \approx \bar{k}^a$ iff $\bar{k}'^a = c\,\bar{k}^a$) such that $\mathcal{L}_{\bar{k}} \bar{\omega}_a = 0$. An NEH equipped with an equivalence class $[\bar{k}^a]$ of null normals with this property is called a *Weakly Isolated Horizon (WIH)*. (Every Killing Horizon is a WIH; but KHs constitute only a small and special sub-class.)

Time Dependence on Geometric WIHs

- On any geometrical WIH $(\mathfrak{h}, [\bar{k}^a], \bar{q}_{ab}, \bar{D})$, fields $\bar{q}_{ab}$, $\bar{\omega}_a$ are time independent :

$$\dot{\bar{q}}_{ab} := \mathcal{L}_{\bar{k}}\, \bar{q}_{ab} = 0 \quad \text{and} \quad \dot{\bar{\omega}}_a := \mathcal{L}_{\bar{k}}\, \bar{\omega}_a = 0.$$

Furthermore, one can show that, given any horizontal 1-form h_a (i.e. $h_a \bar{k}^a = 0$),

$$\dot{\bar{D}}_a h_b := (\mathcal{L}_{\bar{k}} \bar{D}_a - \bar{D}_a \mathcal{L}_{\bar{k}}) h_b = 0.$$

- Thus, time dependence of $\bar{D}$ is completely determined by $\dot{D}_a \bar{j}_b$ for any 1-form $\bar{j}_b$ satisfying $\bar{j}_a \bar{k}^a = -1$. It is given by:

$$\dot{\bar{D}}_a \bar{j}_b = (\bar{D}_a \bar{\omega}_b + \bar{\omega}_a \bar{\omega}_b) + (\underbrace{\bar{k}^c\, \bar{C}_{c\underleftarrow{ab}}{}^d\, \bar{j}_d}_{\text{Weyl}} + \underbrace{\frac{1}{2} \bar{S}_{\underleftarrow{ab}} + f\, \bar{q}_{ab}}_{\text{Ricci}}) \qquad (1).$$

None of the terms on the right hand side vanishes on $\mathfrak{h}$! Thus part of the geometry of a WIH is dynamical. This dynamics is driven by the pull-back to $\mathfrak{h}$ of the 4-d curvature tensor $\bar{R}_{abc}{}^d$ (since $\bar{\omega}_a$ is already a part of $\bar{D}$).

- So far no field equations have been imposed; we only have a geometric condition on $\bar{R}_a{}^b$. $\mathscr{I}^+$ is an asymptotic WIH on which the Weyl part vanishes, while on BH horizons Δ, the Ricci part vanishes. We will find that the diametrically opposite physics of Δ and $\mathscr{I}^+$ emerges from the simple fact!

2. BH WIHs Δ

• Einstein's equations hold on BH WIHs. For simplicity, let me assume source-free equations on $\mathfrak{h}$. To emphasize this new input, we will drop 'bars' over symbols $\bar{g}_{ab}$, $\bar{R}_{abc}{}^{d}$, $\bar{D}$ $\bar{k}^a$, $\bar{j}^a$ and denote $\mathfrak{h}$ by Δ. ((k^a, j^a) are often denoted by (ℓ^a, n^a).)

• Now the Ricci tensor terms in equation (1) for $\dot{D}$ vanish. Furthermore, already on geometric WIHs $\mathfrak{h}$, the the Weyl term that enters the equation (1) for $\dot{D}$ is given by $k^c\, C_{c\underleftarrow{a}b}{}^{d}\, j_d = -(\frac{1}{4}\mathcal{R}\, q_{ab} + D_{[a}\omega_{b]})$ (with $\mathcal{R}$ the 2-d scalar curvature). Therefore,

$$\dot{D}_a j_b = D_{(a}\omega_{b)} + \omega_a\omega_b - \tfrac{1}{4}\mathcal{R}\, q_{ab} \qquad (2)$$

• Thus, even on BH WIHs Δ, the derivative operator D is time-dependent! But this dependence is highly constrained, because the right side of (2) is time independent. (Dot of each term vanishes). Therefore, (q_{ab}, D) on Δ are completely determined by its values on a 2-sphere cross-section; they are 'corner data', representing 'Coulombic fields'. There are no 3-dim degrees of freedom that are hallmarks of radiation. That is why physical observables on Δ –mass, angular momentum, as well as all higher multipole moments– are absolutely conserved.

• Incidentally, $\dot{D} \neq 0 \;\Rightarrow$ that the WIHs are much more general than Killing horizons. Yet, this 'weak isolation' suffices for the zeroth and the first law of BH mechanics to hold Δ.

3. Asymptotic WIH $\mathscr{I}^+$

• A physical space-time (M, g_{ab}) is said to be asymptotically flat at future null infinity if it admits a conformal completion $(\hat{M}, \hat{g}_{ab})$, where $\hat{M} = M \cup \mathscr{I}^+$ is a manifold with a boundary $\mathscr{I}^+$, topologically $\mathbb{S}^2 \times \mathbb{R}$, and $\hat{g}_{ab} = \Omega^2 g_{ab}$ on M s.t.

(i) At $\mathscr{I}^+$, we have $\Omega \doteq 0$ and $\hat{\nabla}_a \Omega \not\doteq 0$; and,

(ii) g_{ab} satisfies Einstein's equations $G_{ab} = 8\pi G\, T_{ab}$, with $\Omega^{-2}\, T_{ab}$ admitting a smooth limit to $\mathscr{I}^+$. Thus $\hat{g}_{ab}$ satisfies the conformal Einstein equations.

($\hat{M}$ and all field thereon now carry a hat to emphasize the fact that we now have a conformal completion of the physical space-time.)

• These conditions imply: (a) $\mathscr{I}^+$ is null with a canonical null normal $\hat{k}^a \doteq \hat{\nabla}^a \Omega$; and, (b) we can always choose Ω such that $\hat{\nabla}_a \hat{k}^b \doteq 0$. This is the standard choice in the literature (where $\hat{k}^a$ is denoted by $\hat{n}^a$). The question is: Is this null sub-manifold $\mathscr{I}^+$ of the conformally completed space-time $(\hat{M}, \hat{g}_{ab})$ a WIH?

• Conformal Einstein's equations at $\mathscr{I}^+$ imply: (1) $\hat{R}_a{}^b \hat{k}^b \propto \hat{k}^a$ at $\mathscr{I}^+$, and, (2) $\theta_{\hat{k}} \doteq 0$. Thus $\mathscr{I}^+$ is an NEH. Also, since $\hat{\nabla}_a \hat{k}^b \equiv \hat{\omega}_a \hat{k}^b = 0$, and $\hat{k}^a$ is nowhere vanishing on $\mathscr{I}^+$, we have $\omega_a \doteq 0$ and hence $\mathcal{L}_{\hat{k}} \omega_a \doteq 0$. Thus, $(\mathscr{I}^+, \hat{k}^a, \hat{q}_{ab}, \hat{D})$ is indeed a WIH in $(\hat{M}, \hat{g}_{ab})$, with $\hat{k}^a \equiv \hat{\nabla}^a \Omega$ as its canonical geodesic null normal.

Time dependence of $\hat{D}$ on $\mathscr{I}^+$

- Since $\mathscr{I}^+$ is a WIH, the metric $\hat{q}_{ab}$ is time independent. Recall that on any geometic WIH, the time dependence of $\bar{D}$ is given by

$$\dot{\bar{D}}_a \bar{j}_b = \bar{D}_a \bar{\omega}_b + \bar{\omega}_a \bar{\omega}_b + \bar{k}^c \, \bar{C}_{c\underleftarrow{ab}}{}^d \, \bar{j}_d \; + \; \tfrac{1}{2} \bar{S}_{\underleftarrow{ab}} + f \, \bar{q}_{ab} \qquad (1).$$

The gray terms (the only ones which were non-zero on the BH WIH Δ) vanish on $\mathscr{I}^+$ and the brown terms (the ones which vanished on the BH WIH Δ) are now non-zero! Thus, at $\mathscr{I}^+$ we have:

$$\dot{\hat{D}}_a \hat{\ell}_b = \tfrac{1}{2} \hat{S}_{\underleftarrow{ab}} + f \, \hat{q}_{ab} \quad \text{or, in a Bondi conformal frame,} \quad \mathrm{TF}\,(\dot{\hat{D}}_a \hat{\ell}_b) = \tfrac{1}{2} \hat{N}_{ab} \quad (3).$$

- While time-dependence of D was driven by the Weyl curvature of g_{ab} at Δ, at $\mathscr{I}^+$ is driven by the Ricci curvature of $\hat{g}_{ab}$, the conformally invariant part of which is just the Bondi news! Therefore, $\hat{D}$ has two-degrees of freedom per point of $\mathscr{I}^+$. They represent freely specified fields on $\mathscr{I}^+$, representing the two radiative modes. Put differently, while the WIH geometry of Δ, carries only 'coulombic' information contained in the 'corner data', that at $\mathscr{I}^+$ carries 'radiative' information contained on all of $\mathscr{I}^+$.

This diametrically opposite physics emerges from the same equation, (1), because Einstein's and conformal Einstein's equations set complementary terms to zero!

4. Symmetries

• Space-time boundaries inherit a 'universal structure' that is shared by all space-times satisfying the boundary conditions. The symmetry group is the sub-group of the diffeomorphism group that preserves the boundary and its universal structure.

• Recall that at $\mathscr{I}^+$ the universal structure consists of pairs $(\hat{q}^\circ_{ab}, \hat{k}^a_\circ)$ –where $\hat{q}^\circ_{ab}$ are unit, round 2-sphere metrics and $\hat{k}^a_\circ$ null normals– 'conformally' related by $(\hat{q}^{\circ\,\prime}_{ab}, \hat{k}'^a_\circ) = (\alpha^2 \hat{q}^\circ_{ab}, \alpha^{-1}\hat{k}^a_\circ)$. The subgroup of the diffeomorphism of $\mathscr{I}^+$ that preserves this stature is the BMS group $\mathfrak{B} = \hat{\mathfrak{s}} \ltimes \mathfrak{L}$, the semi-direct product of the group $\hat{\mathfrak{s}}$ of supertranslations with the Lorentz group $\mathfrak{L}$.

• For general WIHs $\mathfrak{h}$ –as well as for the BH WIHs Δ– the universal structure is the same, except that in place of an unique null normal $\hat{k}^a \hat{=} \hat{\nabla}^a \Omega$ we only have an equivalence class k^a where $k'^a \equiv c\, k^a$ where c is a positive constant on Δ. As a consequence, the symmetry group $\mathfrak{G}$ is simply a 1-dimensional extension of $\mathfrak{B}$: $\mathfrak{G} = \mathfrak{B} \ltimes \mathfrak{D}$ where $\mathfrak{D}$ is the dilation subgroup with action $(q^\circ_{ab}, k^a_\circ) \to (q^\circ_{ab}, c k^a_\circ)$. (This extension is surprising at first but essential because, e.g., the standard time translation in the Schwarzschild space-time is a (super)translation at $\mathscr{I}^+$ but a dilation on its WIH Δ with a geodesic null normal.) The BMS group $\mathfrak{B}$ descends from $\mathfrak{G}$ because $\mathscr{I}^+$ is a WIH with one bit of extra structure.

5. Summary

• Null infinity $\mathscr{I}^+$ of asymptotically flat space-times, and horizons Δ of BHs in equilibrium, are null 3-manifolds but have very different physical connotations. Thus, Δ lies in the strong field region and there is no radiation flux across it, while $\mathscr{I}^+$ lies in the asymptotic, weak field regime with arbitrarily large fluxes of radiation across it! Yet, they share a large number of geometric properties, making them both WIHs $\mathfrak{h}$. Consequently symmetry groups of Δ and $\mathscr{I}^+$ are almost the same. The origin of the drastic differences in their physics is remarkably simple: Einstein's equations hold on Δ, while conformal Einstein's equations hold on $\mathscr{I}^+$!
(All the WIHs considerations apply also to the standard cosmological horizons).

• A standard Hamiltonian framework associates charges and fluxes with all symmetries of of all WIHs $\mathfrak{h}$. It yields the standard BMS charges and fluxes at $\mathscr{I}^+$ and the standard charges (and zero fluxes) at Δ. This unified framework paves a way to explore the relation between horizon dynamics in the strong field region and waveforms at infinity. (AA & Krishnan, arXiv: 2502.11825; Sections V and VI.B.)

Source Material

- References most directly related to the talk:

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2. AA & S. Speziale, Details: PRD 110, 044048 (2024); arXiv: 2402.17977 and PRD 110, 044049 (2024); arXiv:2407.03254

- Further details on Weakly Isolated Horizons:

3. AA, C. Beetle, J. Lewandowski, Mechanics of Rotating Isolated Horizons, Physical Review D64 (2001) 044016; gr-qc/0103026

4. AA, N. Khera, M. Kolanowski and J. Lewandowski, Non-expanding horizons: Multipoles and symmetries, JHEP 01, 28 (2022), arXiv:2111.07873

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- Further details on Null Infinity:

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14. V. Chandrasekaran, E. E. Flanagan, and K. Prabhu, Symmetries and charges of general relativity at null boundaries JHEP 11, 125 (2018), arXiv:1807.11499

- Further details on the Interplay between Horizons and Null Infinity

15. A. Ashtekar and B. Krishnan, Quasi-Local Black Hole Horizons: Recent advances, Living Rev. (Rel) 28, 8 (2025), arXiv: 2502.11825; Sections V and VI.B.

SUPPLEMENTARY MATETIAL

Organization

0. Preamble ✓
1. Geometrical Weakly Isolated Horizons (WIHs) ✓
2. Black Hole (and Cosmological) WIHs ✓
3. Null Infinity as a WIH ✓
4. Fluxes (and Charges) associated with Symmetries

References for the source material are listed at the end.

4A. Symmetries of WIHs

• Let's return to geometrical WIHs $\mathfrak{h}$. Fields they are equipped with, $(\bar{q}_{ab}, [\bar{k}^a])$, vary from one WIH to another. However, each $\mathfrak{h}$ carries a 3-parameter family of round, unit metrics $\mathring{q}_{ab}$, conformally related to its $\bar{q}_{ab}$, hence a 3-parameter family of pairs $(\mathring{q}_{ab} := \mathring{\psi}^2 \bar{q}_{ab}, [\mathring{k}^a] = \mathring{\psi}^{-1} \bar{k}^a)$. (Rescaling of $[\bar{k}^a]$, which is natural rescaling for vectors, plays an essential role in the multipole moment considerations.)

• While the conformal factor $\mathring{\psi}$ in $\mathring{q}_{ab} := \mathring{\psi}^2 \bar{q}_{ab}$ varies from one WIH to another, the relative conformal factors $\mathring{\alpha}$ between two unit round metrics, $\mathring{q}'_{ab} = \mathring{\alpha}^2 \mathring{q}_{ab}$, are universal: (normalized) linear combinations of the first 4 spherical harmonics of $\mathring{q}_{ab}$. Thus, the universal structure of WIHs consists of conformally related pairs $(\mathring{q}_{ab}, [\mathring{k}^a])$, where $(\mathring{q}'_{ab}, [\mathring{k}'^a]) = (\mathring{\alpha}^2 \mathring{q}_{ab}, [\mathring{\alpha}^{-1} \mathring{k}^a])$ on $\mathfrak{h}$.

• The WIH symmetry group $\mathfrak{G}$ is the subgroup of diffeos of $\mathfrak{h}$ that preserve this universal structure. It has a semi-direct product structure: $\mathfrak{G} = \mathfrak{B} \ltimes \mathfrak{D}$, where $\mathfrak{B}$ is the BMS group and $\mathfrak{D}$ is a 1-d group of dilations. Action of the infinitesimal generators is given by: $\mathcal{L}_\xi \, \mathring{q}_{ab} = 2\mathring{\beta} \, \mathring{q}_{ab}$ and $\mathcal{L}_\xi \mathring{k}^a = -(\mathring{\beta} + \mathring{\varpi}) \mathring{k}^a$ where, as in the BMS case, $\mathring{\beta}$ is a linear combination of the $\ell = 1$ spherical harmonics, while the constant $\mathring{\varpi}$ is new, representing the action of dilation which only rescales each $\mathring{k}^a$ in $[\mathring{k}]$ by a constant, leaving $\mathring{q}_{ab}$ unchanged.

Reduction of $\mathfrak{G}$ to $\mathfrak{B}$ on asymptotic WIHs $\mathscr{I}^+$

• So far we consider geometrical WIHs $\mathfrak{h}$. Nothing changes on BH WIHs Δ. The symmetry group is again $\mathfrak{G}$. Could we add fields to the universal structure to eliminate the 1-d dilation group $\mathfrak{D}$ on $\mathfrak{h}$? The answer is in the negative: For example, on the Schwarzschild WIH Δ, the static Killing field is a dilation. So if one were to add structure to eliminate $\mathfrak{D}$, the time translation symmetry would fail to belong to the resulting WIH symmetry group!

• Let us now consider $\mathscr{I}^+$ as a WIH in any given conformal completion $(\hat{M}, \hat{g}_{ab})$. This WIH comes with a preferred null normal $\hat{n}^a \hat{=} \hat{\nabla}^a \Omega$, rather than an equivalence class $[\hat{n}^a]$. Therefore, from the WIH perspective, the universal structure now reduces to the pairs $(\mathring{\hat{q}}'_{ab}, \mathring{\hat{n}}^a) \approx (\mathring{\alpha}^2 \mathring{\hat{q}}'_{ab}, \mathring{\alpha}^{-1} \mathring{\hat{n}}^a)$ –i.e. Bondi conformal frames. The subgroup of Diff$(\mathscr{I}^+)$ that preserves this universal structure is precisely the BMS group $\mathfrak{B}$! The BMS vector fields satisfy $\mathcal{L}_\xi \, \mathring{q}_{ab} = 2\mathring{\beta} \, \mathring{q}_{ab}$ and $\mathcal{L}_\xi \mathring{n}^a = -(\mathring{\beta} + \not{\partial})\mathring{n}^a$

Thus, $\mathfrak{G}$ reduces to $\mathfrak{B}$ because on $\mathscr{I}^+$ we have have a single $\hat{n}^a$ in place of an equivalence class $[k^a]$ we hd on Δ. The dilation disappears. What about the Killing field in, say, Schwarzschild? While it is a dilation on Δ, it is a supertranslation on $\mathscr{I}^+$. Interesting and rather subtle interplay between Δ and $\mathscr{I}^+$ that allows us to treat both in the general WIH framework.

4.B The $\mathfrak{B}$ (BMS) Fluxes at $\mathscr{I}^+$

- The covariant phase space $\mathbf{\Gamma}_{\rm cov}$ of interest: space of asymptotically flat solutions g_{ab} to vacuum equations, naturally equipped with a pre-symplectic structure $\boldsymbol{\omega}$. $\mathbf{\Gamma}_{\rm cov}$ is a rather complicated infinite-dimensional non-linear space and $\boldsymbol{\omega}$ is degenerate precisely along gauge directions.

- Major simplifications occur at $\mathscr{I}^+$:

(i) One can isolate the true degrees of freedom (TDF), thereby eliminating the gauge freedom. These are precisely the connections $\hat{D}$ inherited by $\mathscr{I}^+$ by the fact that it is a WIH in $(\hat{M}, \hat{g}_{ab})$.
(ii) The TDF are freely specifiable, whence associated with any open region $\hat{\mathcal{R}}$ of $\mathscr{I}^+$ bounded by two cross-sections S_1, S_2 we have a radiative phase space $\mathbf{\Gamma}_{\hat{\mathcal{R}}}$ which is now a topological affine space. The induced symplectic structure $\boldsymbol{\omega}_{\hat{\mathcal{R}}}$ is non-degenerate because the TDF are gauge invariant. Thus, at $\mathscr{I}^+$ there are major simplifications & much more mathematical control on relative to $\mathbf{\Gamma}_{\rm cov}$ because $\mathbf{\Gamma}_{\hat{\mathcal{R}}}$ is linear.

- Result: Every BMS vector field ξ^a induces a vector field X_ξ on $\mathbf{\Gamma}_{\hat{\mathcal{R}}}$ which preserves $\boldsymbol{\omega}_{\hat{\mathcal{R}}}$ and therefore defines a Hamiltonian function:

$$H_\xi \,=\, \tfrac{1}{16\pi G}\, \textstyle\int_{\hat{\mathcal{R}}} \left[(\mathcal{L}_\xi \hat{D}_a - \hat{D}_a \mathcal{L}_\xi)\hat{\ell}_b\right] \hat{N}_{cd}\, \hat{q}^{ac}\hat{q}^{bd}\, \mathrm{d}^3\mathscr{I}^+$$

Physically, H_ξ provides the flux of the observables associated with BMS symmetries, i.e., supermomenta and angular momenta. (Their expressions agree with those derived from the standard supermomentum and angular momentum charges (Dray)).

4.C The $\mathfrak{G}$-Fluxes on BH horizons Δ

• What fluxes do we obtain if we apply this procedure to open regions $\mathcal{R}$ of Black hole horizons Δ using infinitesimal generators ξ^a of $\mathfrak{G}$? Space-time metrics induce fields (q_{ab}, D) on Δ (and hence on regions $\mathcal{R}$) that constitute $\mathbf{\Gamma}_{\mathcal{R}}$. They can again be labelled by freely specifiable fields, but now the DOF are only 2-d corner terms!

• Therefore, the same procedure to compute the fluxes H_ξ for $\mathfrak{G}$-symmetries ξ^a now implies that they vanish identically: there are no radiative DOF on Δ. Physically this is just what one expects for horizons representing BHs in equilibrium. But we now see explicitly that one and the same strategy –of using phase spaces that capture the DOF that reside in regions $\hat{\mathcal{R}}$ of $\mathscr{I}^+$ and $\mathcal{R}$ of Δ– yields strikingly different physical results at $\mathscr{I}^+$ and at Δ although both are WIHs!

• Fluxes can be expressed using just the true/radiative DOFs, just as one would physically expect. On the other hand, the 2-sphere charges $Q_\xi[S]$ associated with cross-sections S of $\mathscr{I}^+$ and Δ depend also on Coulombic fields ('corner data') that is not captured in $\mathbf{\Gamma}_{\hat{\mathcal{R}}}$ and $\mathbf{\Gamma}_{\mathcal{R}}$. So we need additional input from full Einstein's equations. This additional input enables one to obtain 2-sphere charges both at $\mathscr{I}^+$ and Δ compatible with fluxes.